22.1 Introduction

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply.

In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in I.C. engines, the energy is developed only during power stroke which is much more than the engine load, and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus
Flywheel

rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.

In machines where the operation is intermittent like punching machines, shearing machines, riveting machines, crushers etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

Note: The function of a governor in engine is entirely different from that of a flywheel. It regulates the mean speed of an engine when there are variations in the load, e.g. when the load on the engine increases, it becomes necessary to increase the supply of working fluid. On the other hand, when the load decreases, less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load condition and keeps the mean speed within certain limits.

As discussed above, the flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation. It does not control the speed variations caused by the varying load.

22.2 Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed. The ratio of the maximum fluctuation of speed to the mean speed is called coefficient of fluctuation of speed.

Let

\[ N_1 = \text{Maximum speed in r.p.m. during the cycle}, \]
\[ N_2 = \text{Minimum speed in r.p.m. during the cycle}, \]
\[ N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2} \]

\[ \therefore \text{Coefficient of fluctuation of speed}, \]

\[ C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2} \]

\[ = \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{(In terms of angular speeds)} \]

\[ = \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{(In terms of linear speeds)} \]

The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It varies depending upon the nature of service to which the flywheel is employed. Table 22.1 shows the permissible values for coefficient of fluctuation of speed for some machines.

Note: The reciprocal of coefficient of fluctuation of speed is known as coefficient of steadiness and it is denoted by \( m \).

\[ \therefore \]

\[ m = \frac{1}{C_S} = \frac{N}{N_1 - N_2} = \frac{\omega}{\omega_1 - \omega_2} = \frac{v}{v_1 - v_2} \]
Table 22.1. Permissible values for coefficient of fluctuation of speed ($C_s$).

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Type of machine or class of service</th>
<th>Coefficient of fluctuation of speed ($C_s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Crushing machines</td>
<td>0.200</td>
</tr>
<tr>
<td>2.</td>
<td>Electrical machines</td>
<td>0.003</td>
</tr>
<tr>
<td>3.</td>
<td>Electrical machines (direct drive)</td>
<td>0.002</td>
</tr>
<tr>
<td>4.</td>
<td>Engines with belt transmission</td>
<td>0.030</td>
</tr>
<tr>
<td>5.</td>
<td>Gear wheel transmission</td>
<td>0.020</td>
</tr>
<tr>
<td>6.</td>
<td>Hammering machines</td>
<td>0.200</td>
</tr>
<tr>
<td>7.</td>
<td>Pumping machines</td>
<td>0.03 to 0.05</td>
</tr>
<tr>
<td>8.</td>
<td>Machine tools</td>
<td>0.030</td>
</tr>
<tr>
<td>9.</td>
<td>Paper making, textile and weaving machines</td>
<td>0.025</td>
</tr>
<tr>
<td>10.</td>
<td>Punching, shearing and power presses</td>
<td>0.10 to 0.15</td>
</tr>
<tr>
<td>11.</td>
<td>Spinning machinery</td>
<td>0.10 to 0.020</td>
</tr>
<tr>
<td>12.</td>
<td>Rolling mills and mining machines</td>
<td>0.025</td>
</tr>
</tbody>
</table>

22.3 Fluctuation of Energy

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider a turning moment diagram for a single cylinder double acting steam engine as shown in Fig. 22.1. The vertical ordinate represents the turning moment and the horizontal ordinate (abscissa) represents the crank angle.

A little consideration will show that the turning moment is zero when the crank angle is zero. It rises to a maximum value when crank angle reaches 90° and it is again zero when crank angle is 180°. This is shown by the curve $abc$ in Fig. 22.1 and it represents the turning moment diagram for outstroke. The curve $cde$ is the turning moment diagram for instroke and is somewhat similar to the curve $abc$.

Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution. In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line $AF$. The height of the ordinate $aA$ represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle $aAFe$ is proportional to the work done against the mean resisting torque.

![Fig. 22.1. Turning moment diagram for a single cylinder double acting steam engine.](image-url)
the area $aBp$, whereas the energy required is represented by the area $aABp$. In other words, the engine has done less work (equal to the area $aAB$) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases. Now the crank moves from $p$ to $q$, the work done by the engine is equal to the area $pBbCq$, whereas the requirement of energy is represented by the area $pBcq$. Therefore the engine has done more work than the requirement. This excess work (equal to the area $BbC$) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from $p$ to $q$.

Similarly when the crank moves from $q$ to $r$, more work is taken from the engine than is developed. This loss of work is represented by the area $CcD$. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from $q$ to $r$. As the crank moves from $r$ to $s$, excess energy is again developed given by the area $DdE$ and the speed again increases. As the piston moves from $s$ to $e$, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called fluctuation of energy. The areas $BbC$, $CcD$, $DdE$ etc. represent fluctuations of energy.

A turning moment diagram for a four stroke internal combustion engine is shown in Fig. 22.2. We know that in a four stroke internal combustion engine, there is one working stroke after the crank has turned through $720^\circ$ (or $4\pi$ radians). Since the pressure inside the engine cylinder is less than the atmospheric pressure during suction stroke, therefore a negative loop is formed as shown in Fig. 22.2. During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained. In the working stroke, the fuel burns and the gases expand, therefore a large positive loop is formed. During exhaust stroke, the work is done on the gases, therefore a negative loop is obtained.

A turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in Fig. 22.3. The resultant turning moment diagram is the sum of
the turning moment diagrams for the three cylinders. It may be noted that the first cylinder is the high pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder. The cranks, in case of three cylinders are usually placed at 120º to each other.

22.4 Maximum Fluctuation of Energy

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. 22.4. The horizontal line AG represents the mean torque line. Let \( a_1, a_2, a_3 \) be the areas above the mean torque line and \( a_2, a_4 \) and \( a_6 \) be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

Let the energy in the flywheel at \( A = E \), then from Fig. 22.4, we have

- Energy at \( B = E + a_1 \)
- Energy at \( C = E + a_1 - a_2 \)
- Energy at \( D = E + a_1 - a_2 + a_3 \)
- Energy at \( E = E + a_1 - a_2 + a_3 - a_4 \)
- Energy at \( F = E + a_1 - a_2 + a_3 - a_4 + a_5 \)
- Energy at \( G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6 = E \) = Energy at \( A \)

Let us now suppose that the maximum of these energies is at \( B \) and minimum at \( E \).

\[ \therefore \text{Maximum energy in the flywheel} \]
\[ = E + a_1 \]

and minimum energy in the flywheel

\[ = E + a_1 - a_2 + a_3 - a_4 \]
Maximum fluctuation of energy,

\[ \Delta E = \text{Maximum energy} - \text{Minimum energy} \]

\[ = (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4 \]

### 22.5 Coefficient of Fluctuation of Energy

It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It is usually denoted by \( C_E \). Mathematically, coefficient of fluctuation of energy,

\[ C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}} \]

The workdone per cycle may be obtained by using the following relations:

1. Workdone / cycle \( = T_{\text{mean}} \times \theta \)

   where \( T_{\text{mean}} = \text{Mean torque, and} \)

   \( \theta = \text{Angle turned in radians per revolution} \)

   \( = 2\pi, \text{ in case of steam engines and two stroke internal combustion engines.} \)

   \( = 4\pi, \text{ in case of four stroke internal combustion engines.} \)

   The mean torque \( (T_{\text{mean}}) \) in N-m may be obtained by using the following relation \( i.e. \)

   \[ T_{\text{mean}} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega} \]

   where \( P = \text{Power transmitted in watts,} \)

   \( N = \text{Speed in r.p.m., and} \)

   \( \omega = \text{Angular speed in rad/s} = \frac{2\pi N}{60} \)

2. The workdone per cycle may also be obtained by using the following relation:

   \[ \text{Workdone / cycle} = \frac{P \times 60}{n} \]

   where \( n = \text{Number of working strokes per minute.} \)

   \( = N, \text{ in case of steam engines and two stroke internal combustion engines.} \)

   \( = N/2, \text{ in case of four stroke internal combustion engines.} \)

   The following table shows the values of coefficient of fluctuation of energy for steam engines and internal combustion engines.

   **Table 22.2. Coefficient of fluctuation of energy \( (C_E) \) for steam and internal combustion engines.**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Type of engine</th>
<th>Coefficient of fluctuation of energy ( (C_E) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Single cylinder, double acting steam engine</td>
<td>0.21</td>
</tr>
<tr>
<td>2.</td>
<td>Cross-compound steam engine</td>
<td>0.096</td>
</tr>
<tr>
<td>3.</td>
<td>Single cylinder, single acting, four stroke gas engine</td>
<td>1.93</td>
</tr>
<tr>
<td>4.</td>
<td>Four cylinder, single acting, four stroke gas engine</td>
<td>0.066</td>
</tr>
<tr>
<td>5.</td>
<td>Six cylinder, single acting, four stroke gas engine</td>
<td>0.031</td>
</tr>
</tbody>
</table>

### 22.6 Energy Stored in a Flywheel

A flywheel is shown in Fig. 22.5. We have already discussed that when a flywheel absorbs energy its speed increases and when it gives up energy its speed decreases.
Let

\[ m = \text{Mass of the flywheel in kg}, \]
\[ k = \text{Radius of gyration of the flywheel in metres}, \]
\[ I = \text{Mass moment of inertia of the flywheel about the axis of rotation in kg-m}^2 = m.k^2, \]
\[ N_1 \text{ and } N_2 = \text{Maximum and minimum speeds during the cycle in r.p.m.}, \]
\[ \omega_1 \text{ and } \omega_2 = \text{Maximum and minimum angular speeds during the cycle in rad / s}, \]
\[ N = \text{Mean speed during the cycle in r.p.m.} = \frac{N_1 + N_2}{2}, \]
\[ \omega = \text{Mean angular speed during the cycle in rad / s} = \frac{\omega_1 + \omega_2}{2} \]
\[ C_S = \text{Coefficient of fluctuation of speed} = \frac{N_1 - N_2}{N} \text{ or } \frac{\omega_1 - \omega_2}{\omega} \]

We know that mean kinetic energy of the flywheel,
\[ E = \frac{1}{2} \times I \omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \text{ (in N-m or joules)} \]

As the speed of the flywheel changes from \( \omega_1 \) to \( \omega_2 \), the maximum fluctuation of energy,
\[ \Delta E = \text{Maximum K.E. — Minimum K.E.} = \frac{1}{2} \times I (\omega_1)^2 - \frac{1}{2} \times I (\omega_2)^2 \]
\[ = \frac{1}{2} \times I \left[ (\omega_1)^2 - (\omega_2)^2 \right] = \frac{1}{2} \times I \left( \omega_1 + \omega_2 \right) (\omega_1 - \omega_2) \]
\[ = I.\omega (\omega_1 - \omega_2) \quad \text{...} \quad \omega = \frac{\omega_1 + \omega_2}{2} \quad \text{...(i)} \]
\[ = I.\omega^2 \left( \frac{\omega_1 - \omega_2}{\omega} \right) \quad \text{...}[\text{Multiplying and dividing by } \omega] \]
\[ = I.\omega^2.C_S = m.k^2.\omega^2.C_S \quad \text{...(ii)} \]
\[ = 2 \times E.C_S \quad \text{...} \quad E = \frac{1}{2} \times I.\omega^2 \quad \text{...(iii)} \]

The radius of gyration \( k \) may be taken equal to the mean radius of the rim \( R \), because the thickness of rim is very small as compared to the diameter of rim. Therefore substituting \( k = R \) in equation \( (ii) \), we have
\[ \Delta E = m.R^2.\omega^2.C_S = m.\nu^2.C_S \quad \text{...(\( \therefore \nu = \omega.R \))} \]

From this expression, the mass of the flywheel rim may be determined.

**Notes:**
1. In the above expression, only the mass moment of inertia of the rim is considered and the mass moment of inertia of the hub and arms is neglected. This is due to the fact that the major portion of weight of the flywheel is in the rim and a small portion is in the hub and arms. Also the hub and arms are nearer to the axis of rotation, therefore the moment of inertia of the hub and arms is very small.
2. The density of cast iron may be taken as 7260 kg / m³ and for cast steel, it may taken as 7800 kg / m³.
3. The mass of the flywheel rim is given by
\[ m = \text{Volume} \times \text{Density} = 2 \pi R \times A \times \rho \]
From this expression, we may find the value of the cross-sectional area of the rim. Assuming the cross-section of the rim to be rectangular, then

\[ A = b \times t \]

where

- \( b \) = Width of the rim, and
- \( t \) = Thickness of the rim.

Knowing the ratio of \( b/t \) which is usually taken as 2, we may find the width and thickness of rim.

4. When the flywheel is to be used as a pulley, then the width of rim should be taken 20 to 40 mm greater than the width of belt.

**Example 22.1.** The turning moment diagram for a petrol engine is drawn to the following scales:

**Turning moment**, 1 mm = 5 N-m;
**Crank angle**, 1 mm = 1°.

The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line, taken in order are 295, 685, 40, 340, 960, 270 mm².

Determine the mass of 300 mm diameter flywheel rim when the coefficient of fluctuation of speed is 0.3% and the engine runs at 1800 r.p.m. Also determine the cross-section of the rim when the width of the rim is twice of thickness. Assume density of rim material as 7250 kg / m³.

**Solution.** Given : \( D = 300 \) mm or \( R = 150 \) mm = 0.15 m; \( C_s = 0.3\% = 0.003 \); \( N = 1800 \) r.p.m. or \( \omega = 2 \pi \times 1800 / 60 = 188.5 \) rad/s; \( \rho = 7250 \) kg / m³

**Mass of the flywheel**

Let \( m \) = Mass of the flywheel in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig. 22.6.

Since the scale of turning moment is 1 mm = 5 N-m, and scale of the crank angle is 1 mm = 1° = \( \pi / 180 \) rad, therefore 1 mm² on the turning moment diagram.

\[ = 5 \times \pi / 180 = 0.087 \text{ N-m} \]

Let the total energy at \( A = E \). Therefore from Fig. 22.6, we find that

- Energy at \( B = E + 295 \)
- Energy at \( C = E + 295 - 685 = E - 390 \)
- Energy at \( D = E - 390 + 40 = E - 350 \)
- Energy at \( E = E - 350 - 340 = E - 690 \)
- Energy at \( F = E - 690 + 960 = E + 270 \)
- Energy at \( G = E + 270 - 270 = E = \text{Energy at } A \)

From above we see that the energy is maximum at \( B \) and minimum at \( E \).

\[ \therefore \text{ Maximum energy } = E + 295 \]

and minimum energy \( = E - 690 \)
We know that maximum fluctuation of energy,
\[ \Delta E = \text{Maximum energy} - \text{Minimum energy} \]
\[ = (E + 295) - (E - 690) = 985 \text{ mm}^2 \]
\[ = 985 \times 0.087 \times 10^{-9} = 86 \text{ N} \cdot \text{m} \]

We also know that maximum fluctuation of energy (\( \Delta E \)),
\[ 86 = m.R^2.\omega^2.C_S = m (0.15)^2 (188.5)^2 (0.003) = 2.4 \text{ m} \]
\[ \therefore \quad m = \frac{86}{2.4} = 35.8 \text{ kg} \quad \text{Ans.} \]

**Cross-section of the flywheel rim**

Let \( t \) = Thickness of rim in metres, and \( b \) = Width of rim in metres = 2 \( t \) ...(Given)

\[ \therefore \quad \text{Cross-sectional area of rim}, \quad A = b \times t = 2t \times t = 2t^2 \]

We know that mass of the flywheel rim (\( m \)),
\[ 35.8 = A \times 2\pi R \times \rho = 2t^2 \times 2\pi \times 0.15 \times 7250 = 13 \, 668 \, t^2 \]
\[ \therefore \quad t^2 = \frac{35.8}{13 \, 668} = 0.0026 \quad \text{or} \quad t = 0.051 \text{ m} = 51 \text{ mm} \quad \text{Ans.} \]

and \( b = 2t = 2 \times 51 = 102 \text{ mm} \quad \text{Ans.} \)

**Example 22.2.** The intercepted areas between the output torque curve and the mean resistance line of a turning moment diagram for a multi-cylinder engine, taken in order from one end are as follows:

\[ -35, +410, -285, +325, -335, +260, -365, +285, -260 \text{ mm}^2. \]

The diagram has been drawn to a scale of 1 mm = 70 N-m and 1 mm = 4.5°. The engine speed is 900 r.p.m. and the fluctuation in speed is not to exceed 2% of the mean speed.

Find the mass and cross-section of the flywheel rim having 650 mm mean diameter. The density of the material of the flywheel may be taken as 7200 kg / m³. The rim is rectangular with the width 2 times the thickness. Neglect effect of arms, etc.

**Solution.** Given: \( N = 900 \text{ r.p.m.} \) or \( \omega = 2\pi \times 900 / 60 = 94.26 \text{ rad/s} \); \( \omega_1 - \omega_2 = 2\% \ \omega \) or \( \frac{\omega_2 - \omega_1}{\omega} = C_S = 2\% = 0.02 \); \( D = 650 \text{ mm} \) or \( R = 325 \text{ mm} \); \( \rho = 7200 \text{ kg} / \text{ m}^3 \)

**Mass of the flywheel rim**

Let \( m = \text{Mass of the flywheel rim in kg.} \)

First of all, let us find the maximum fluctuation of energy. The turning moment diagram for a multi-cylinder engine is shown in Fig. 22.7. Since the scale of turning moment is 1 mm = 70 N-m and scale of the crank angle is 1 mm = 4.5° = \( \pi / 40 \) rad, therefore 1 mm² on the turning moment diagram.

\[ = 70 \times \pi / 40 = 5.5 \text{ N-m} \]
Let the total energy at \( A = E \). Therefore from Fig. 22.7, we find that

Energy at \( B = E - 35 \)

Energy at \( C = E - 35 + 410 = E + 375 \)

Energy at \( D = E + 375 - 285 = E + 90 \)

Energy at \( E = E + 90 + 325 = E + 415 \)

Energy at \( F = E + 415 - 335 = E + 80 \)

Energy at \( G = E + 80 + 260 = E + 340 \)

Energy at \( H = E + 340 - 365 = E - 25 \)

Energy at \( K = E - 25 + 285 = E + 260 \)

Energy at \( L = E + 260 - 260 = E = \) Energy at \( A \)

From above, we see that the energy is maximum at \( E \) and minimum at \( B \).

\[ \therefore \text{Maximum energy} = E + 415 \]

and

\[ \text{minimum energy} = E - 35 \]

We know that maximum fluctuation of energy,

\[ (E + 415) - (E - 35) = 450 \text{ mm}^2 \]

\[ = 450 \times 5.5 = 2475 \text{ N-m} \]

We also know that maximum fluctuation of energy (\( \Delta E \)),

\[ 2475 = mR^2 \omega^2 C_\xi = m (0.325)^2 (94.26)^2 0.02 = 18.77 \text{ m} \]

\[ \therefore m = \frac{2475}{18.77} = 132 \text{ kg} \text{ Ans.} \]

**Cross-section of the flywheel rim**

Let

\[ t = \text{Thickness of the rim in metres, and} \]

\[ b = \text{Width of the rim in metres} = 2t \]

...(Given)

\[ \therefore \text{Area of cross-section of the rim,} \]

\[ A = b \times t = 2t \times t = 2t^2 \]

We know that mass of the flywheel rim (\( m \)),

\[ 132 = A \times 2 \pi R \times \rho = 2 t^2 \times 2 \pi \times 0.325 \times 7200 = 29409 t^2 \]

\[ \therefore t^2 = \frac{132}{29409} = 0.0044 \quad \text{or} \quad t = 0.067 \text{ m} = 67 \text{ mm Ans.} \]

and

\[ b = 2t = 2 \times 67 = 134 \text{ mm Ans.} \]

**Example 22.3.** A single cylinder double acting steam engine develops 150 kW at a mean speed of 80 r.p.m. The coefficient of fluctuation of energy is 0.1 and the fluctuation of speed is \( \pm 2\% \) of mean speed. If the mean diameter of the flywheel rim is 2 metres and the hub and spokes provide 5 percent of the rotational inertia of the wheel, find the mass of the flywheel and cross-sectional area of the rim. Assume the density of the flywheel material (which is cast iron) as 7200 kg / m³.
Solution. Given: \( P = 150 \text{ kW} = 150 \times 10^3 \text{ W} \); \( N = 80 \text{ r.p.m.} \); \( C_E = 0.1 \); \( \omega_1 - \omega_2 = \pm 2\% \omega \); \( D = 2 \text{ m} \) or \( R = 1 \text{ m} \); \( \rho = 7200 \text{ kg/m}^3 \)

**Mass of the flywheel rim**

Let \( m \) = Mass of the flywheel rim in kg.

We know that the mean angular speed,

\[
\omega = \frac{2 \pi N}{60} = \frac{2 \pi \times 80}{60} = 8.4 \text{ rad/s}
\]

Since the fluctuation of speed is \( \pm 2\% \) of mean speed (\( \omega \)), therefore total fluctuation of speed,

\[
\omega_1 - \omega_2 = 4 \% \omega = 0.04 \omega
\]

and coefficient of fluctuation of speed,

\[
C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.04
\]

We know that the work done by the flywheel per cycle

\[
= \frac{P \times 60}{N} = \frac{150 \times 10^3 \times 60}{80} = 112500 \text{ N-m}
\]

We also know that coefficient of fluctuation of energy,

\[
C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Workdone / cycle}}
\]

\[
= \frac{0.1 \times 112500}{112500} = 0.1\times 112500 = 11250 \text{ N-m}
\]

Since 5\% of the rotational inertia is provided by hub and spokes, therefore the maximum fluctuation of energy of the flywheel rim will be 95\% of the flywheel.

\[
(\Delta E)_{\text{rim}} = 0.95 \times 112500 = 10687.5 \text{ N-m}
\]

We know that maximum fluctuation of energy of the rim \( (\Delta E)_\text{rim} \),

\[
10687.5 = mR^2 \omega^2 C_S = m \times 1^2 (8.4)^2 \times 0.04 = 2.82 m
\]

\[
\therefore \ m = 10687.5 / 2.82 = 3790 \text{ kg} \ \text{Ans.}
\]

**Cross-sectional area of the rim**

Let \( A = \) Cross-sectional area of the rim.

We know that the mass of the flywheel rim \( (m) \),

\[
3790 = A \times 2 \pi R \times \rho = A \times 2 \pi \times 1 \times 7200 = 45245 A
\]

\[
\therefore \ A = 3790 / 45245 = 0.084 \text{ m}^2 \ \text{Ans.}
\]

**Example 22.4.** A single cylinder, single acting, four stroke oil engine develops 20 kW at 300 r.p.m. The work done by the gases during the expansion stroke is 2.3 times the work done on the gases during the compression and the work done during the suction and exhaust strokes is negligible. The speed is to be maintained within \( \pm 1\% \). Determine the mass moment of inertia of the flywheel.

Solution. Given: \( P = 20 \text{ kW} = 20 \times 10^3 \text{ W} \); \( N = 300 \text{ r.p.m.} \) or \( \omega = 2 \pi \times 300 / 60 = 31.42 \text{ rad/s} \); \( \omega_1 - \omega_2 = \pm 1\% \omega \)

First of all, let us find the maximum fluctuation of energy \( (\Delta E) \). The turning moment diagram for a four stroke engine is shown in Fig. 22.8. It is assumed to be triangular during compression and expansion strokes, neglecting the suction and exhaust strokes.
We know that mean torque transmitted by the engine,

\[ T_{mean} = \frac{P \times 60}{2 \pi N} = \frac{20 \times 10^3 \times 60}{2 \pi \times 300} = 636.5 \text{ N-m} \]

and \( \text{work done per cycle} = T_{mean} \times \theta = 636.5 \times 4 \pi = 8000 \text{ N-m} \) \( \text{...(i)} \)

Let \( W_C = \text{Work done during compression stroke} \), and \( W_E = \text{Work done during expansion stroke} \).

**Fig. 22.8**

Since the work done during suction and exhaust strokes is negligible, therefore net work done per cycle

\[ = W_E - W_C = W_E - W_E / 2.3 = 0.565 W_E \] \( \text{...(ii)} \)

From equations (i) and (ii), we have

\[ W_E = 8000 / 0.565 = 14160 \text{ N-m} \]

The work done during the expansion stroke is shown by triangle \( ABC \) in Fig. 22.8, in which base \( AC = \pi \) radians and height \( BF = T_{max} \).

\[ \therefore \text{Work done during expansion stroke} (W_E), \]

\[ 14160 = \frac{1}{2} \times \pi \times T_{max} = 1.571 T_{max} \]

or

\[ T_{max} = 14160 / 1.571 = 9013 \text{ N-m} \]

We know that height above the mean torque line,

\[ BG = BF - FG = T_{max} - T_{mean} \]

\[ = 9013 - 636.5 = 8376.5 \text{ N-m} \]

Since the area \( BDE \) shown shaded in Fig. 22.8 above the mean torque line represents the maximum fluctuation of energy \( (\Delta E) \), therefore from geometrical relation,

\[ \frac{\text{Area of } \Delta BDE}{\text{Area of } \Delta ABC} = \frac{(BG)^2}{(BF)^2}, \text{ we have} \]

* The work done per cycle may also be calculated as follows:

We know that for a four stroke engine, number of working strokes per cycle

\[ n = N / 2 = 300 / 2 = 150 \]

\[ \therefore \text{Work done per cycle} = P \times 60 / n = 20 \times 10^3 \times 60 / 150 = 8000 \text{ N-m} \]
A Textbook of Machine Design

Maximum fluctuation of energy (i.e. area of $\Delta BDE$),

\[
\Delta E = \text{Area of } \Delta ABC \left( \frac{BG}{BF} \right)^2 = W_E \left( \frac{BG}{BF} \right)^2
\]

\[
= 14 \times 160 \left( \frac{8376.5}{9013} \right)^2 = 12230 \text{ N-m}
\]

Since the speed is to be maintained within $\pm 1\%$ of the mean speed, therefore total fluctuation of speed

\[
\omega_1 - \omega_2 = 2\% \quad \omega = 0.02 \omega
\]

and coefficient of fluctuation of speed,

\[
C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.02
\]

Let

\[ I = \text{Mass moment of inertia of the flywheel in kg-m}^2. \]

We know that maximum fluctuation of energy ($\Delta E$),

\[ 12230 = I \omega^2 C_S \]

\[ = I (31.42)^2 0.02 = 19.74 I \]

\[ \therefore \quad I = 12230 / 19.74 = 619.5 \text{ kg-m}^2 \quad \text{Ans.} \]

22.7 Stresses in a Flywheel Rim

A flywheel, as shown in Fig. 22.9, consists of a rim at which the major portion of the mass or weight of flywheel is concentrated, a boss or hub for fixing the flywheel on to the shaft and a number of arms for supporting the rim on the hub.

The following types of stresses are induced in the rim of a flywheel:

1. Tensile stress due to centrifugal force,
2. Tensile bending stress caused by the restraint of the arms, and
3. The shrinkage stresses due to unequal rate of cooling of casting. These stresses may be very high but there is no easy method of determining. This stress is taken care of by a factor of safety.

We shall now discuss the first two types of stresses as follows:

1. **Tensile stress due to the centrifugal force**

   The tensile stress in the rim due to the centrifugal force, assuming that the rim is unstrained by the arms, is determined in a similar way as a thin cylinder subjected to internal pressure.

   Let
   
   \[ b = \text{Width of rim}, \]
   \[ t = \text{Thickness of rim}, \]

   \* The maximum fluctuation of energy ($\Delta E$) may also be obtained as discussed below:

   From similar triangles $BDE$ and $BAC$,

   \[
   \frac{DE}{AC} = \frac{BG}{BF} \quad \text{or} \quad DE = \frac{BG}{BF} \times AC = \frac{8376.5}{9013} \times \pi = 2.92 \text{ rad}
   \]

   \[ \therefore \quad \text{Maximum fluctuation of energy (i.e. area of } \Delta BDE), \]

   \[
   \Delta E = \frac{1}{2} \times DE \times BG = \frac{1}{2} \times 2.92 \times 8376.5 = 12230 \text{ N-m}
   \]
Consider a small element of the rim as shown shaded in Fig. 22.10. Let it subtend an angle $\delta \theta$ at the centre of the flywheel.

Volume of the small element
\[ = A \cdot R \cdot \delta \theta \]

\[ \therefore \text{Mass of the small element,} \]
\[ dm = \text{Volume} \times \text{Density} = A \cdot R \cdot \delta \theta \cdot \rho = \rho \cdot A \cdot R \cdot \delta \theta \]

and centrifugal force on the element,
\[ dF = dm \cdot \omega^2 \cdot R = \rho \cdot A \cdot R^2 \cdot \omega^2 \cdot \delta \theta \]

Vertical component of $dF$
\[ = dF \cdot \sin \theta = \rho \cdot A \cdot R^2 \cdot \omega^2 \cdot \delta \theta \cdot \sin \theta \]

\[ \therefore \text{Total vertical bursting force across the rim diameter X-Y,} \]
\[ = \rho \cdot A \cdot R^2 \cdot \omega^2 \int_0^\pi \sin \theta d\theta \]
\[ = \rho \cdot A \cdot R^2 \cdot \omega^2 \left[ - \cos \theta \right]_0^\pi = 2 \rho \cdot A \cdot R^2 \cdot \omega^2 \]

... (i)

This vertical force is resisted by a force of $2P$, such that
\[ 2P = 2\sigma_t \times A \]

... (ii)

From equations (i) and (ii), we have
\[ 2\rho A R^2 \cdot \omega^2 = 2 \sigma_t \times A \]

\[ \therefore \sigma_t = \rho \cdot R^2 \cdot \omega^2 = \rho \cdot v^2 \]

... (iii)

when $\rho$ is in kg / m$^3$ and $v$ is in m / s, then $\sigma_t$ will be in N / m$^2$ or Pa.
A Textbook of Machine Design

Note: From the above expression, the mean diameter \((D)\) of the flywheel may be obtained by using the relation,

\[ v = \frac{\pi D N}{60} \]

2. Tensile bending stress caused by restraint of the arms

The tensile bending stress in the rim due to the restraint of the arms is based on the assumption that each portion of the rim between a pair of arms behaves like a beam fixed at both ends and uniformly loaded, as shown in Fig. 22.11, such that length between fixed ends,

\[ l = \frac{\pi D}{n} = \frac{2 \pi R}{n} \]

where \(n\) = Number of arms.

The uniformly distributed load \((w)\) per metre length will be equal to the centrifugal force between a pair of arms.

\[ w = b . t . \rho \cdot \omega^2 . R \text{ N/m} \]

We know that maximum bending moment,

\[ M = \frac{w . l^2}{12} = \frac{b . t . \rho \cdot \omega^2 \cdot R \left(\frac{2 \pi R}{n}\right)^2}{12} \]

and section modulus,

\[ Z = \frac{1}{6} b \times t^2 \]

Fig. 22.11

\[ \therefore \text{Bending stress,} \]

\[ \sigma_b = \frac{M}{Z} = \frac{b \cdot t \cdot \rho \cdot \omega^2 \cdot R \left(\frac{2 \pi R}{n}\right)^2}{12} \times \frac{6}{b \times t^2} \]

\[ = \frac{19.74 \cdot \rho \cdot \omega^2 \cdot R^3}{n^2 \cdot t} = \frac{19.74 \cdot \rho \cdot v^2 \cdot R}{n^2 \cdot t} \]

\[ \text{...(iv)} \]

... (Substituting \(\omega = v/R\))

Now total stress in the rim,

\[ \sigma = \sigma_t + \sigma_b \]

If the arms of a flywheel do not stretch at all and are placed very close together, then centrifugal force will not set up stress in the rim. In other words, \(\sigma_t\) will be zero. On the other hand, if the arms are stretched enough to allow free expansion of the rim due to centrifugal action, there will be no restraint due to the arms, \(i.e. \sigma_b\) will be zero.

It has been shown by G. Lanza that the arms of a flywheel stretch about \(\frac{3}{4}\) th of the amount necessary for free expansion. Therefore the total stress in the rim,

\[ \sigma = \frac{3}{4} \sigma_t + \frac{1}{4} \sigma_b = \frac{3}{4} \rho \cdot v^2 + \frac{1}{4} \times \frac{19.74 \cdot \rho \cdot v^2 \cdot R}{n^2 \cdot t} \]

\[ \text{...(v)} \]

\[ = \rho \cdot v^2 \left(0.75 + \frac{4.935 R}{n^2 \cdot t}\right) \]
Example 22.5. A multi-cylinder engine is to run at a constant load at a speed of 600 r.p.m. On drawing the crank effort diagram to a scale of 1 m = 250 N-m and 1 mm = 3°, the areas in sq mm above and below the mean torque line are as follows:

+ 160, – 172, + 168, – 191, + 197, – 162 sq mm

The speed is to be kept within ± 1% of the mean speed of the engine. Calculate the necessary moment of inertia of the flywheel.

Determine suitable dimensions for cast iron flywheel with a rim whose breadth is twice its radial thickness. The density of cast iron is 7250 kg / m³, and its working stress in tension is 6 MPa. Assume that the rim contributes 92% of the flywheel effect.

Solution. Given: \( N = 600 \) r.p.m. or \( \omega = 2\pi \times 600 / 60 = 62.84 \) rad / s; \( \rho = 7250 \) kg / m³; \( \sigma_t = 6 \) MPa = \( 6 \times 10^6 \) N/m²

Moment of inertia of the flywheel

Let \( I = \) Moment of inertia of the flywheel.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig. 22.12.

Since the scale for the turning moment is 1 mm = 250 N-m and the scale for the crank angle is

1 mm = 3° = \( \frac{\pi}{60} \) rad, therefore

1 mm² on the turning moment diagram

\( = 250 \times \frac{\pi}{60} = 13.1 \) N-m

Let the total energy at \( A = E \). Therefore from Fig. 22.12, we find that

Energy at \( B = E + 160 \)
Energy at \( C = E + 160 – 172 = E – 12 \)
Energy at \( D = E – 12 + 168 = E + 156 \)
Energy at \( E = E + 156 – 191 = E – 35 \)
Energy at \( F = E – 35 + 197 = E + 162 \)
Energy at \( G = E + 162 - 162 = E = \text{Energy at } A \)

From above, we find that the energy is maximum at \( F \) and minimum at \( E \).

\[ \therefore \text{Maximum energy} = E + 162 \]

and minimum energy = \( E - 35 \)

We know that the maximum fluctuation of energy,

\[ \Delta E = \text{Maximum energy} - \text{Minimum energy} \]

\[ = (E + 162) - (E - 35) = 197 \text{ mm}^2 = 197 \times 13.1 = 2581 \text{ N-m} \]

Since the fluctuation of speed is \( \pm 1\% \) of the mean speed \( \omega \), therefore total fluctuation of speed,

\[ \omega_1 - \omega_2 = 2\% \omega = 0.02 \omega \]

and coefficient of fluctuation of speed,

\[ C_s = \frac{\omega_1 - \omega_2}{\omega} = 0.02 \]

We know that the maximum fluctuation of energy \( (\Delta E) \),

\[ 2581 = I \omega^2 C_s = I (62.84)^2 0.02 = 79 I \]

\[ \therefore I = 2581 / 79 = 32.7 \text{ kg-m}^2 \text{ Ans.} \]

**Dimensions of a flywheel rim**

Let \( t \) = Thickness of the flywheel rim in metres, and

\( b = \text{Breadth of the flywheel rim in metres} = 2t \) \( \text{...(Given)} \)

First of all let us find the peripheral velocity \( (v) \) and mean diameter \( (D) \) of the flywheel.

We know that tensile stress \( (\sigma_t) \),

\[ 6 \times 10^6 = \rho v^2 = 7250 \times v^2 \]

\[ \therefore v^2 = 6 \times 10^6 / 7250 = 827.6 \quad \text{or} \quad v = 28.76 \text{ m/s} \]

We also know that peripheral velocity \( (v) \),

\[ 28.76 = \pi D / 60 = \pi D \times 600 / 60 = 31.42 D \]

\[ \therefore D = 28.76 / 31.42 = 0.915 \text{ m} = 915 \text{ mm} \text{ Ans.} \]

Now let us find the mass of the flywheel rim. Since the rim contributes \( 92\% \) of the flywheel effect, therefore the energy of the flywheel rim \( (E_{rim}) \) will be \( 0.92 \) times the total energy of the flywheel \( (E) \). We know that maximum fluctuation of energy \( (\Delta E) \),

\[ 2581 = E \times 2 C_s = E \times 2 \times 0.02 = 0.04 E \]

\[ \therefore E = 2581 / 0.04 = 64525 \text{ N-m} \]

and energy of the flywheel rim,

\[ E_{rim} = 0.92 E = 0.92 \times 64525 = 59363 \text{ N-m} \]

Let \( m \) = Mass of the flywheel rim.

We know that energy of the flywheel rim \( (E_{rim}) \),

\[ 59363 = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times m \times (28.76)^2 = 413.6 m \]

\[ \therefore m = 59363 / 413.6 = 143.5 \text{ kg} \]

We also know that mass of the flywheel rim \( (m) \),

\[ 143.5 = b \times t \times \pi D \times \rho = 2t \times t \times \pi \times 0.915 \times 7250 = 41686 t^2 \]
\[ r^2 = \frac{143.5}{41686} = 0.00344 \]

or
\[ t = 0.0587 \text{ say } 0.06 \text{ m} = 60 \text{ mm \, Ans.} \]

and
\[ b = 2t = 2 \times 60 = 120 \text{ mm \, Ans.} \]

Notes: The mass of the flywheel rim may also be obtained by using the following relations. Since the rim contributes 92% of the flywheel effect, therefore using

1. \[ I_{rim} = 0.92 I_{flywheel} \quad \text{or} \quad m.k^2 = 0.92 \times 32.7 = 30 \text{ kg-m}^2 \]

Since radius of gyration, \( k = \frac{R}{2} = \frac{0.915}{2} = 0.4575 \text{ m}, \) therefore
\[ m = \frac{30}{k^2} = \frac{30}{(0.4575)^2} = \frac{30}{0.209} = 143.5 \text{ kg} \]

2. \( (\Delta E)_{rim} = 0.92 (\Delta E)_{flywheel} \]
\[ m.v^2C_S = 0.92 (\Delta E)_{flywheel} \]
\[ m(28.76)^20.02 = 0.92 	imes 2581 \]
\[ 16.55m = 2374.5 \quad \text{or} \quad m = 2374.5 / 16.55 = 143.5 \text{ kg} \]

Example 22.6. The areas of the turning moment diagram for one revolution of a multi-cylinder engine with reference to the mean turning moment, below and above the line, are

\(-32, +408, -267, +333, -310, +226, -374, +260 \text{ and } -244 \text{ mm}^2.\)

The scale for abscissa and ordinate are: 1 mm = 2.4° and 1 mm = 650 N-m respectively. The mean speed is 300 r.p.m. with a percentage speed fluctuation of ±1.5%. If the hoop stress in the material of the rim is not to exceed 5.6 MPa, determine the suitable diameter and cross-section for the flywheel, assuming that the width is equal to 4 times the thickness. The density of the material may be taken as 7200 kg / m³. Neglect the effect of the boss and arms.
Solution. Given: \( N = 300 \) r.p.m. or \( \omega = 2 \pi \times 300/60 = 31.42 \) rad/s; \( \sigma_t = 5.6 \) MPa = \( 5.6 \times 10^6 \) N/m\(^2\); \( \rho = 7200 \) kg/m\(^3\)

**Diameter of the flywheel**

Let \( D \) = Diameter of the flywheel in metres.

We know that peripheral velocity of the flywheel,
\[
v = \frac{\pi D N}{60} = \frac{\pi D \times 300}{60} = 15.71 \text{ D m/s}
\]

We also know that hoop stress \( (\sigma_t) \),
\[
5.6 \times 10^6 = \rho \times v^2 = 7200 (15.71 D)^2 = 1.8 \times 10^6 D^2
\]
\[
\therefore \quad D^2 = 5.6 \times 10^6 / 1.8 \times 10^6 = 3.11 \quad \text{or} \quad D = 1.764 \text{ m} \quad \text{Ans.}
\]

**Cross-section of the flywheel**

Let 
\( t \) = Thickness of the flywheel rim in metres, and 
\( b = Width \) of the flywheel rim in metres = \( 4 t \) \( ...(\text{Given}) \)
\[
\therefore \quad \text{Cross-sectional area of the rim,} \quad A = b \times t = 4 t \times t = 4 t^2 \text{ m}^2
\]

Now let us find the maximum fluctuation of energy. The turning moment diagram for one revolution of a multi-cylinder engine is shown in Fig. 22.13.

![Fig. 22.13](image-url)

Since the scale of crank angle is \( 1 \text{ mm} = 2.4^\circ = 2.4 \times \frac{\pi}{180} = 0.042 \text{ rad} \), and the scale of the turning moment is \( 1 \text{ mm} = 650 \text{ N-m} \), therefore

\[1 \text{ mm}^2 \text{ on the turning moment diagram} = 650 \times 0.042 = 27.3 \text{ N-m}\]

Let the total energy at \( A = E \). Therefore from Fig. 22.13, we find that

Energy at \( B = E - 32 \)
Energy at \( C = E - 32 + 408 = E + 376 \)
Energy at \( D = E + 376 - 267 = E + 109 \)
Energy at \( E = E + 109 + 333 = E + 442 \)
Energy at \( F = E + 442 - 310 = E + 132 \)
Energy at $G = E + 132 + 226 = E + 358$
Energy at $H = E + 358 - 374 = E - 16$
Energy at $I = E - 16 + 260 = E + 244$
Energy at $J = E + 244 - 244 = E = \text{Energy at } A$

From above, we see that the energy is maximum at $E$ and minimum at $B$.

∴ Maximum energy $= E + 442$
and minimum energy $= E - 32$

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 442) - (E - 32) = 474 \text{ mm}^2$$

Since the fluctuation of speed is $\pm 1.5\%$ of the mean speed, therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 3\% \text{ of mean speed} = 0.03 \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.03$$

Let $m$ = Mass of the flywheel rim.

We know that maximum fluctuation of energy ($\Delta E$),

$$12,940 = m.R^2.\omega^2.C_S = m \left(\frac{1.764}{2}\right)^2 (31.42)^2 0.03 = 23 m$$

∴ $m = 12,940 / 23 = 563 \text{ kg Ans.}$

We also know that mass of the flywheel rim ($m$),

$$563 = A \times \pi D \times \rho = 4 t^2 \times \pi \times 1.764 \times 7200 = 159,624 t^2$$

∴ $t = 0.0594 \text{ m} = 59.4 \text{ say 60 mm Ans.}$

and $b = 4t = 4 \times 60 = 240 \text{ mm Ans.}$

Example 22.7. An otto cycle engine develops 50 kW at 150 r.p.m. with 75 explosions per minute. The change of speed from the commencement to the end of power stroke must not exceed $0.5\%$ of mean on either side. Design a suitable rim section having width four times the depth so that the hoop stress does not exceed 4 MPa. Assume that the flywheel stores 16/15 times the energy stored by the rim and that the work done during power stroke is 1.40 times the work done during the cycle. Density of rim material is 7200 kg / m$^3$.

Solution. Given : $P = 50 \text{ kW} = 50 \times 10^3 \text{ W} ; N = 150 \text{ r.p.m.} ; n = 75 ; \sigma = 4 \text{ MPa} = 4 \times 10^6 \text{ N/m}^2 ; \rho = 7200 \text{ kg/m}^3$.

First of all, let us find the mean torque ($T_{\text{mean}}$) transmitted by the engine or flywheel. We know that the power transmitted ($P$),

$$50 \times 10^3 = \frac{2 \pi N \times T_{\text{mean}}}{60} = 15.71 \times T_{\text{mean}}$$

∴ $T_{\text{mean}} = 50 \times 10^3 / 15.71 = 3182.7 \text{ N-m}$

Since the explosions per minute are equal to $N/2$, therefore the engine is a four stroke cycle engine. The turning moment diagram of a four stroke engine is shown in Fig. 22.14.
We know that \[ \text{work done per cycle} = T_{\text{mean}} \times \theta = 3182.7 \times 4 \pi = 40000 \text{ N-m} \]

\[ \therefore \text{work done during power or working stroke} = 1.4 \times 40000 = 56000 \text{ N-m} \]...

(b) The work done during power or working stroke is shown by a triangle ABC in Fig. 22.14 in which base AC = \(\pi\) radians and height BF = \(T_{\text{max}}\).

\[ \therefore \text{work done during working stroke} = \frac{1}{2} \times \pi \times T_{\text{max}} = 1.571 T_{\text{max}} \]...

From equations (i) and (ii), we have

\[ T_{\text{max}} = \frac{56000}{1.571} = 35646 \text{ N-m} \]

Height above the mean torque line,

\[ BG = BF - FG = T_{\text{max}} - T_{\text{mean}} = 35646 - 3182.7 = 32463.3 \text{ N-m} \]

Since the area BDE (shown shaded in Fig. 22.14) above the mean torque line represents the maximum fluctuation of energy (\(\Delta E\)), therefore from geometrical relation

\[ \frac{\text{Area of } \Delta BDE}{\text{Area of } \Delta ABC} = \frac{(BG)^2}{(BF)^2}, \text{ we have} \]

Maximum fluctuation of energy (i.e. area of triangle BDE),

\[ \Delta E = \text{Area of triangle } ABC \times \left( \frac{BG}{BF} \right)^2 = 56000 \times \left( \frac{32463.3}{35646} \right)^2 \]

\[ = 56000 \times 0.83 = 46480 \text{ N-m} \]

**Mean diameter of the flywheel**

Let \(D\) = Mean diameter of the flywheel in metres, and \(v\) = Peripheral velocity of the flywheel in m/s.

* The work done per cycle for a four stroke engine is also given by

\[ \text{Work done / cycle} = \frac{P \times 60}{\text{Number of explosion / min}} = \frac{P \times 60}{n} = \frac{50000 \times 60}{75} = 40000 \text{ N-m} \]
We know that hoop stress ($\sigma_t$),
$$4 \times 10^6 = \rho v^2 = 7200 \times v^2$$
∴ $v = 23.58$ m/s

We also know that peripheral velocity ($v$),
$$23.58 = \pi \frac{D N}{60} = \frac{\pi D \times 150}{60} = 7.855 D$$
∴ $D = 23.58 / 7.855 = 3$ m \textbf{Ans.}

**Cross-sectional dimensions of the rim**

Let $t =$Thickness of the rim in metres, and
$b =$ Width of the rim in metres $= 4 t$ ...(Given)
∴ Cross-sectional area of the rim,
$$A = b \times t = 4 t \times t = 4 t^2$$

First of all, let us find the mass of the flywheel rim.

Let $m =$ Mass of the flywheel rim, and
$E =$ Total energy of the flywheel.

Since the fluctuation of speed is 0.5% of the mean speed on either side, therefore total fluctuation of speed,
$$N_1 - N_2 = 1\% \text{ of mean speed} = 0.01 N$$
and coefficient of fluctuation of speed,
$$C_S = \frac{N_1 - N_2}{N} = 0.01$$

We know that the maximum fluctuation of energy ($\Delta E$),
$$46 480 = E \times 2 C_S = E \times 2 \times 0.01 = 0.02 E$$
∴ $E = 46 480 / 0.02 = 2324 \times 10^3$ N-m

Since the energy stored by the flywheel is $\frac{16}{15}$ times the energy stored by the rim, therefore the energy of the rim,
$$E_{rim} = \frac{15}{16} E = \frac{15}{16} \times 2324 \times 10^3 = 2178.8 \times 10^3$ N-m

We know that energy of the rim ($E_{rim}$),
$$2178.8 \times 10^3 = \frac{1}{2} m \times v^2 = \frac{1}{2} m (23.58)^2 = 278 m$$
∴ $m = 2178.8 \times 10^3 / 278 = 7837$ kg

We also know that mass of the flywheel rim ($m$),
$$7837 = A \times \pi D \times \rho = 4 t^2 \times \pi \times 3 \times 7200 = 271 469 t^2$$
∴ $t^2 = 7837 / 271 469 = 0.0288$ or $t = 0.17$ m $= 170$ mm \textbf{Ans.}

and
$$b = 4 t = 4 \times 170 = 680$$ mm \textbf{Ans.}

\textbf{Example 22.8.} A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during 1 / 2 revolution and remains constant for the following revolution. It then falls uniformly to 750 N-m during the next 1 / 2 revolution and remains constant for one revolution, the cycle being repeated thereafter. Determine the power required to drive the machine.
If the total fluctuation of speed is not to exceed 3% of the mean speed, determine a suitable diameter and cross-section of the flywheel rim. The width of the rim is to be 4 times the thickness and the safe centrifugal stress is 6 MPa. The material density may be assumed as 7200 kg/m³.

Solution. Given:

\[ N = 250 \text{ r.p.m. or } \omega = 2\pi \times 250 / 60 = 26.2 \text{ rad/s} ; \omega_1 - \omega_2 = 3\% \omega \]

\[ C_S = 3\% = 0.03 \]

\[ \sigma_t = 6 \text{ MPa} = 6 \times 10^6 \text{ N/m}^2 \]

\[ \rho = 7200 \text{ kg/m}^3 \]

Power required to drive the machine

The turning moment diagram for the complete cycle is shown in Fig. 22.15.

![Fig. 22.15](image)

We know that the torque required for one complete cycle

\[ = \text{Area of figure } OABCDEF \]

\[ = \text{Area } OAEF + \text{Area } ABG + \text{Area } BCHG + \text{Area } CDH \]

\[ = OF \times OA + \frac{1}{2} \times AG \times BG + GH \times CH + \frac{1}{2} \times HD \times CH \]

\[ = 6\pi \times 750 + \frac{1}{2} \times \pi \times (3000 - 750) + 2\pi (3000 - 750) + \frac{1}{2} \times \pi (3000 - 750) \]

\[ = 4500\pi + 1125\pi + 4500\pi + 1125\pi = 11250\pi \text{ N-m} \quad \ldots (i) \]

If \( T_{\text{mean}} \) is the mean torque in N-m, then torque required for one complete cycle

\[ = T_{\text{mean}} \times 6 \pi \text{ N-m} \quad \ldots (ii) \]

From equations (i) and (ii),

\[ T_{\text{mean}} = 11250\pi / 6\pi = 1875 \text{ N-m} \]

We know that power required to drive the machine,

\[ P = T_{\text{mean}} \times \omega = 1875 \times 26.2 = 49125 \text{ W} = 49.125 \text{ kW} \quad \text{Ans.} \]

Diameter of the flywheel

Let

\[ D = \text{Diameter of the flywheel in metres, and} \]

\[ v = \text{Peripheral velocity of the flywheel in m/s.} \]

We know that the centrifugal stress \( \sigma_t \),

\[ 6 \times 10^6 = \rho \times v^2 = 7200 \times v^2 \]

\[ \therefore \quad v^2 = 6 \times 10^6 / 7200 = 833.3 \quad \text{or} \quad v = 28.87 \text{ m/s} \]
We also know that peripheral velocity of the flywheel \( (v) \),
\[
28.87 = \frac{\pi D N}{60} = \frac{\pi D \times 250}{60} = 13.1 D
\]
\[
\therefore \quad D = 28.87 / 13.1 = 2.2 \text{ m} \quad \text{Ans.}
\]

**Cross-section of the flywheel rim**

Let
\[
t = \text{Thickness of the flywheel rim in metres, and}
\]
\[
b = \text{Width of the flywheel rim in metres} = 4 \times t
\]
\[\text{...(Given)}\]
\[
\therefore \quad \text{Cross-sectional area of the flywheel rim},
\]
\[
A = b \times t = 4 \times t \times t = 4 \times t^2 \text{ m}^2
\]

First of all, let us find the maximum fluctuation of energy \((\Delta E)\) and mass of the flywheel rim \((m)\). In order to find \(\Delta E\), we shall calculate the values of \(LM\) and \(NP\).

From similar triangles \(ABG\) and \(BLM\),
\[
\frac{LM}{AG} = \frac{BM}{BG} \quad \text{or} \quad \frac{LM}{\pi} = \frac{3000 - 1857}{3000 - 750} = 0.5 \quad \text{or} \quad LM = 0.5 \pi
\]

Now from similar triangles \(CHD\) and \(CNP\),
\[
\frac{NP}{HD} = \frac{CN}{CH} \quad \text{or} \quad \frac{NP}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad \text{or} \quad NP = 0.5 \pi
\]

From Fig. 22.15, we find that
\[
BM = CN = 3000 - 1875 = 1125 \text{ N-m}
\]

Since the area above the mean torque line represents the maximum fluctuation of energy, therefore maximum fluctuation of energy,
\[
\Delta E = \text{Area of } LBCP = \text{Area of } LBM + \text{Area of } MBCN + \text{Area of } PNC
\]
\[
= \frac{1}{2} \times LM \times BM + MN \times BM + \frac{1}{2} \times NP \times CN
\]
\[
= \frac{1}{2} \times 0.5 \pi \times 1125 + 2 \pi \times 1125 + \frac{1}{2} \times 0.5 \pi \times 1125
\]
\[
= 8837 \text{ N-m}
\]

We know that maximum fluctuation of energy \((\Delta E)\),
\[
8837 = m.R^2.\omega^2.C_s = m \left( \frac{2.2}{2} \right)^2 (26.2)^2 0.03 = 24.9 m
\]
\[
\therefore \quad m = 8837 / 24.9 = 355 \text{ kg}
\]

We also know that mass of the flywheel rim \((m)\),
\[
355 = A \times \pi D \times \rho = 4 \times \pi \times 2.2 \times 7200 = 199 \text{ 077 lb}
\]
\[
\therefore \quad \rho = 355 / 199 \text{ 077} = 0.00178 \text{ or } t = 0.042 \text{ m} = 42 \text{ mm} \quad \text{Ans.}
\]

and
\[
b = 4 \times t = 4 \times 45 = 180 \text{ mm} \quad \text{Ans.}
\]

**Example 22.9.** A punching machine makes 25 working strokes per minute and is capable of punching 25 mm diameter holes in 18 mm thick steel plates having an ultimate shear strength of 300 MPa.

The punching operation takes place during 1/10 th of a revolution of the crank shaft.

Estimate the power needed for the driving motor, assuming a mechanical efficiency of 95 per cent. Determine suitable dimensions for the rim cross-section of the flywheel, which is to revolve at 9 times the speed of the crank shaft. The permissible coefficient of fluctuation of speed is 0.1.
A Textbook of Machine Design

The flywheel is to be made of cast iron having a working stress (tensile) of 6 MPa and density of 7250 kg/m³. The diameter of the flywheel must not exceed 1.4 m owing to space restrictions. The hub and the spokes may be assumed to provide 5% of the rotational inertia of the wheel.

Check for the centrifugal stress induced in the rim.

Solution. Given: 
\[ n = 25 \; ; \; d_1 = 25 \text{ mm} \; ; \; t_1 = 18 \text{ mm} \; ; \; \tau_u = 300 \text{ MPa} = 300 \text{ N/mm}^2 \; ; \; \eta_m = 95\% = 0.95 \; ; \; C_S = 0.1 \; ; \; \sigma_t = 6 \text{ MPa} = 6 \text{ N/mm}^2 \; ; \; \rho = 7250 \text{ kg/m}^3 \; ; \; D = 1.4 \text{ m} \text{ or } R = 0.7 \text{ m}

Power needed for the driving motor

We know that the area of plate sheared,
\[ A_S = \pi d_1 t_1 = \pi \times 25 \times 18 = 1414 \text{ mm}^2 \]

∴ Maximum shearing force required for punching,
\[ F_S = A_S \tau_u = 1414 \times 300 = 424200 \text{ N} \]

and energy required per stroke
\[ = \frac{1}{2} F_s t_1 = \frac{1}{2} \times 424200 \times 18 = 3817.8 \times 10^3 \text{ N-mm} \]

∴ Energy required per min
\[ = \text{Energy / stroke} \times \text{No. of working strokes / min} \]
\[ = 3817.8 \times 10^3 \times 25 = 95.45 \times 10^6 \text{ N-mm} = 95450 \text{ N-m} \]

We know that the power needed for the driving motor
\[ = \frac{\text{Energy required per min}}{60 \times \eta_m} = \frac{95450}{60 \times 0.95} = 1675 \text{ W} \]

= 1.675 kW Ans.

* As the hole is punched, it is assumed that the shearing force decreases uniformly from maximum value to zero.
**Dimensions for the rim cross-section**

Considering the cross-section of the rim as rectangular and assuming the width of rim equal to twice the thickness of rim.

Let \( t \) = Thickness of rim in metres, and
\( b = 2t \) = Width of rim in metres = \( 2t \).

\[ A = b \times t = 2t \times t = 2t^2 \]

Since the punching operation takes place (i.e. energy is consumed) during 1/10 th of a revolution of the crank shaft, therefore during 9/10 th of the revolution of a crank shaft, the energy is stored in the flywheel.

\[ \Delta E = \frac{9}{10} \times \text{Energy/stroke} = \frac{9}{10} \times 3817.8 \times 10^3 \]
\[ = 3436 \times 10^3 \text{ N-mm} = 3436 \text{ N-m} \]

Let \( m = \text{Mass of the flywheel}. \)

Since the hub and the spokes provide 5% of the rotational inertia of the wheel, therefore the maximum fluctuation of energy provided by the flywheel rim will be 95%.

\[ (\Delta E)_{\text{rim}} = 0.95 \times \Delta E = 0.95 \times 3436 = 3264 \text{ N-m} \]

Since the flywheel is to revolve at 9 times the speed of the crankshaft and there are 25 working strokes per minute, therefore mean speed of the flywheel,
\[ N = 9 \times 25 = 225 \text{ r.p.m.} \]

and mean angular speed,
\[ \omega = 2 \pi \times 225 / 60 = 23.56 \text{ rad/s} \]

We know that maximum fluctuation of energy (\( \Delta E \)),
\[ 3264 = m.R^2.\omega^2.C_S = m (0.7)^2 (23.56)^2 0.1 = 27.2 m \]

\[ m = 3264 / 27.2 = 120 \text{ kg} \]

We also know that mass of the flywheel (m),
\[ 120 = A \times \pi D \times \rho = 2t^2 \times \pi \times 1.4 \times 7250 = 63 \times 782 \times t^2 \]

\[ t^2 = 120 / 63 \times 782 = 0.001 \, \text{Ans.} \text{ or } t = 0.044 \, \text{m} = 44 \, \text{mm} \text{ Ans.} \]

\[ b = 2 \times t = 2 \times 44 = 88 \, \text{mm} \text{ Ans.} \]

**Check for centrifugal stress**

We know that peripheral velocity of the rim,
\[ v = \frac{\pi D \times N}{60} = \frac{\pi \times 1.4 \times 225}{60} = 16.5 \, \text{m/s} \]

\[ \sigma_t = \rho,v^2 = 7250 (16.5)^2 = 1.97 \times 10^6 \, \text{N/m}^2 = 1.97 \, \text{MPa} \]

Since the centrifugal stress induced in the rim is less than the permissible value (i.e. 6 MPa), therefore it is safe Ans.

**22.8 Stresses in Flywheel Arms**

The following stresses are induced in the arms of a flywheel.

1. Tensile stress due to centrifugal force acting on the rim.
2. Bending stress due to the torque transmitted from the rim to the shaft or from the shaft to the rim.
3. Shrinkage stresses due to unequal rate of cooling of casting. These stresses are difficult to determine.
We shall now discuss the first two types of stresses as follows:

1. **Tensile stress due to the centrifugal force**

   Due to the centrifugal force acting on the rim, the arms will be subjected to direct tensile stress whose magnitude is same as discussed in the previous article.

   \[ \sigma_{t1} = \frac{3}{4} \sigma_t = \frac{3}{4} \rho \times v^2 \]

2. **Bending stress due to the torque transmitted**

   Due to the torque transmitted from the rim to the shaft or from the shaft to the rim, the arms will be subjected to bending, because they are required to carry the full torque load. In order to find out the maximum bending moment on the arms, it may be assumed as a cantilever beam fixed at the hub and carrying a concentrated load at the free end of the rim as shown in Fig. 22.16.

   Let
   \[ T = \text{Maximum torque transmitted by the shaft}, \]
   \[ R = \text{Mean radius of the rim}, \]
   \[ r = \text{Radius of the hub}, \]
   \[ n = \text{Number of arms}, \]
   \[ Z = \text{Section modulus for the cross-section of arms}. \]

   We know that the load at the mean radius of the rim,
   \[ F = \frac{T}{R} \]

   \[ \therefore \text{Load on each arm} = \frac{T}{R \times n} \]

   and maximum bending moment which lies on the arm at the hub,
   \[ M = \frac{T}{R \times n} (R - r) \]

   \[ \therefore \text{Bending stress in arms}, \]
   \[ \sigma_{b1} = \frac{M}{Z} = \frac{T}{R \times n \times Z} (R - r) \]

   \[ \therefore \text{Total tensile stress in the arms at the hub end}, \]
   \[ \sigma = \sigma_{t1} + \sigma_{b1} \]

**Notes:**

1. The total stress on the arms should not exceed the allowable permissible stress.
2. If the flywheel is used as a belt pulley, then the arms are also subjected to bending due to net belt tension \( (T_1 - T_2) \), where \( T_1 \) and \( T_2 \) are the tensions in the tight side and slack side of the belt respectively. Therefore the bending stress due to the belt tensions,
   \[ \sigma_{b2} = \frac{(T_1 - T_2) (R - r)}{n \times Z} \]

   \[ \ldots \text{ (\because Only half the number of arms are considered to be effective in transmitting the belt tensions)} \]

   \[ \therefore \text{Total bending stress in the arms at the hub end}, \]
   \[ \sigma_b = \sigma_{b1} + \sigma_{b2} \]

   and the total tensile stress in the arms at the hub end,
   \[ \sigma = \sigma_{t1} + \sigma_{b1} + \sigma_{b2} \]
22.9 Design of Flywheel Arms

The cross-section of the arms is usually elliptical with major axis as twice the minor axis, as shown in Fig. 22.17, and it is designed for the maximum bending stress.

Let
\[ a_1 = \text{Major axis, and} \]
\[ b_1 = \text{Minor axis.} \]

∴ Section modulus,
\[ Z = \frac{\pi}{32} \times b_1 (a_1)^2 \]  \hspace{1cm} \text{...(i)}

We know that maximum bending moment,
\[ M = \frac{T}{R \cdot n} (R - r) \]

∴ Maximum bending stress,
\[ \sigma_b = \frac{M}{Z} = \frac{T}{R \cdot n \cdot Z} (R - r) \]  \hspace{1cm} \text{...(ii)}

Assuming \( a_1 = 2b_1 \), the dimensions of the arms may be obtained from equations (i) and (ii).

Notes:
1. The arms of the flywheel have a taper from the hub to the rim. The taper is about 20 mm per metre length of the arm for the major axis and 10 mm per metre length for the minor axis.
2. The number of arms are usually 6. Sometimes the arms may be 8, 10 or 12 for very large size flywheels.
3. The arms may be curved or straight. But straight arms are easy to cast and are lighter.
4. Since arms are subjected to reversal of stresses, therefore a minimum factor of safety 8 should be used. In some cases like punching machines and machines subjected to severe shock, a factor of safety 15 may be used.
5. The smaller flywheels (less than 600 mm diameter) are not provided with arms. They are made web type with holes in the web to facilitate handling.

22.10 Design of Shaft, Hub and Key

The diameter of shaft for flywheel is obtained from the maximum torque transmitted. We know that the maximum torque transmitted,
\[ T_{\text{max}} = \frac{\pi}{16} \times \tau (d_1)^3 \]

where
\[ d_1 = \text{Diameter of the shaft, and} \]
\[ \tau = \text{Allowable shear stress for the material of the shaft.} \]

The hub is designed as a hollow shaft, for the maximum torque transmitted. We know that the maximum torque transmitted,
\[ T_{\text{max}} = \frac{\pi}{16} \times \tau \left( \frac{d^4 - d_1^4}{d^3} \right) \]

where
\[ d = \text{Outer diameter of hub, and} \]
\[ d_1 = \text{Inner diameter of hub or diameter of shaft.} \]

The diameter of hub is usually taken as twice the diameter of shaft and length from 2 to 2.5 times the shaft diameter. It is generally taken equal to width of the rim.

A standard sunk key is used for the shaft and hub. The length of key is obtained by considering the failure of key in shearing. We know that torque transmitted by shaft,
\[ T_{\text{max}} = L \times w \times \tau \times \frac{d_1}{2} \]

where
\[ L = \text{Length of the key,} \]
\[ \tau = \text{Shear stress for the key material, and} \]
\[ d_1 = \text{Diameter of shaft.} \]
Example 22.10. Design and draw a cast iron flywheel used for a four stroke I.C engine developing 180 kW at 240 r.p.m. The hoop or centrifugal stress developed in the flywheel is 5.2 MPa, the total fluctuation of speed is to be limited to 3% of the mean speed. The work done during the power stroke is 1/3 more than the average work done during the whole cycle. The maximum torque on the shaft is twice the mean torque. The density of cast iron is 7220 kg/m³.

Solution. Given: \( P = 180 \text{ kW} = 180 \times 10^3 \text{ W} \);
\( N = 240 \text{ r.p.m.} \); \( \sigma_t = 5.2 \text{ MPa} = 5.2 \times 10^6 \text{ N/m}^2 \);
\( N_1 - N_2 = 3\% \, N \); \( \rho = 7220 \text{ kg/m}^3 \).

First of all, let us find the maximum fluctuation of energy (\( \Delta E \)). The turning moment diagram of a four stroke engine is shown in Fig. 22.18.

We know that mean torque transmitted by the flywheel,
\[
T_{\text{mean}} = \frac{P \times 60}{2 \pi N} = \frac{180 \times 10^3 \times 60}{2 \pi \times 240} = 7161 \text{ N-m}
\]
and
\[
\text{work done per cycle} = T_{\text{mean}} \times \theta = 7161 \times 4\pi = 90 \, 000 \text{ N-m}
\]

Since the work done during the power stroke is 1/3 more than the average work done during the whole cycle, therefore,

Work done during the power (or working) stroke
\[
= 90 \, 000 + \frac{1}{3} \times 90 \, 000 = 120 \, 000 \text{ N-m} \quad \ldots \ldots \text{(i)}
\]

The work done during the power stroke is shown by a triangle \( ABC \) in Fig. 22.18 in which the base \( AC = \pi \) radians and height \( BF = T_{\text{max}} \).

---

* The work done per cycle may also be obtained as discussed below:

Work done per cycle = \( \frac{P \times 60}{n} \), where \( n = \) Number of working strokes per minute

For a four stroke engine, \( n = \frac{N}{2} = \frac{240}{2} = 120 \)

\[
\therefore \text{Work done per cycle} = \frac{180 \times 10^3 \times 60}{120} = 90 \, 000 \text{ N-m}
\]
\[ \text{Work done during power stroke} = \frac{1}{2} \times \pi \times T_{\text{max}} \]  ... (ii)

From equations (i) and (ii), we have
\[ \frac{1}{2} \times \pi \times T_{\text{max}} = 120,000 \]

\[ \therefore \quad T_{\text{max}} = \frac{120,000 \times 2}{\pi} = 76,384 \text{ N-m} \]

Height above the mean torque line,
\[ BG = BF - FG = T_{\text{max}} - T_{\text{mean}} = 76,384 - 7161 = 69,223 \text{ N-m} \]

Since the area \(\Delta BDE\) shown shaded in Fig. 22.18 above the mean torque line represents the maximum fluctuation of energy \(\Delta E\), therefore from geometrical relation,

\[ \frac{\text{Area of } \Delta BDE}{\text{Area of } \Delta ABC} = \left(\frac{BG}{BF}\right)^2 \]

*Maximum fluctuation of energy (i.e. area of \(\Delta BDE\)),

\[ \Delta E = \text{Area of } \Delta ABC \times \left(\frac{BG}{BF}\right)^2 = 120,000 \left(\frac{69,223}{76,384}\right)^2 = 98,555 \text{ N-m} \]

1. **Diameter of the flywheel rim**

Let \(D\) = Diameter of the flywheel rim in metres, and \(v\) = Peripheral velocity of the flywheel rim in m/s.

We know that the hoop stress developed in the flywheel rim \(\sigma_t\),
\[ 5.2 \times 10^6 = \rho_v^2 = 7,220 \times v^2 \]

\[ \therefore \quad v^2 = \frac{5.2 \times 10^6}{7,220} = 720 \quad \text{or} \quad v = 26.8 \text{ m/s} \]

We also know that peripheral velocity \(v\),
\[ 26.8 = \frac{\pi D \times N}{60} = \frac{2D \times 250}{60} = 13.1 \]

\[ \therefore \quad D = 26.8 / 13.1 = 2.04 \text{ m} \quad \text{Ans.} \]

2. **Mass of the flywheel rim**

Let \(m\) = Mass of the flywheel rim in kg.

We know that angular speed of the flywheel rim,
\[ \omega = \frac{2 \pi N}{60} = \frac{2 \pi \times 250}{60} = 25.14 \text{ rad/s} \]

and coefficient of fluctuation of speed,
\[ C_S = \frac{N_1 - N_2}{N} = 0.03 \]

We know that maximum fluctuation of energy \(\Delta E\),
\[ 98,555 = m \cdot R^2 \cdot \omega^2 \cdot C_S = m \left(\frac{2.04}{2}\right)^2 (25.14)^2 0.03 = 19.73 \text{ m} \]

\[ \therefore \quad m = 98,555 / 19.73 = 4,995 \text{ kg} \quad \text{Ans.} \]

* The approximate value of maximum fluctuation of energy may be obtained as discussed below:

Work done per cycle = 90,000 N-mm ... (as calculated above)

Work done per stroke = 90,000 / 4 = 22,500 N-m ...(\text{of four stroke engine})

and work done during power stroke = 120,000 N-m

\[ \therefore \quad \text{Maximum fluctuation of energy,} \]
\[ \Delta E = 120,000 - 22,500 = 97,500 \text{ N-m} \]
3. Cross-sectional dimensions of the rim

Let

- \( t \) = Depth or thickness of the rim in metres, and
- \( b \) = Width of the rim in metres = \( 2t \) ...(Assume)

∴ Cross-sectional area of the rim,

\[ A = b.t = 2t \times t = 2t^2 \]

We know that mass of the flywheel rim \((m)\),

\[ 4995 = A \times \pi D \times \rho = 2t^2 \times \pi \times 2.04 \times 7220 = 92556t^2 \]

∴ \( t^2 = 4995 / 92556 = 0.054 \) or \( t = 0.232 \) say 0.235 m = 235 mm Ans.

and

\[ b = 2t = 2 \times 235 = 470 \text{ mm Ans.} \]

4. Diameter and length of hub

Let

- \( d \) = Diameter of the hub,
- \( d_1 \) = Diameter of the shaft, and
- \( l \) = Length of the hub.

Since the maximum torque on the shaft is twice the mean torque, therefore maximum torque acting on the shaft,

\[ T_{\text{max}} = 2 \times T_{\text{mean}} = 2 \times 7161 = 14322 \text{ N-m} = 14322 \times 10^3 \text{ N-mm} \]

We know that the maximum torque acting on the shaft \((T_{\text{max}})\),

\[ 14322 \times 10^3 = \frac{\pi}{16} \times \tau (d_1)^3 = \frac{\pi}{16} \times 40 (d_1)^3 = 7.855 (d_1)^3 \]

...(Taking \( \tau = 40 \text{ MPa} = 40 \text{ N/mm}^2 \))

∴ \( (d_1)^3 = 14322 \times 10^3 / 7.855 = 1823 \times 10^3 \)

or

\[ d_1 = 122 \] say 125 m Ans.

The diameter of the hub is made equal to twice the diameter of shaft and length of hub is equal to width of the rim.

∴ \( d = 2d_1 = 2 \times 125 = 250 \) mm = 0.25 m

and

\[ l = b = 470 \text{ mm} = 0.47 \text{ mm Ans.} \]

5. Cross-sectional dimensions of the elliptical arms

Let

- \( a_1 \) = Major axis,
- \( b_1 \) = Minor axis = 0.5 \( a_1 \) ...(Assume)
- \( n \) = Number of arms = 6 ...(Assume)
- \( \sigma_b \) = Bending stress for the material of arms = 15 MPa = 15 N/mm\(^2\) ...(Assume)

We know that the maximum bending moment in the arm at the hub end, which is assumed as cantilever is given by

\[ M = \frac{T}{R.n} (R - r) = \frac{T}{D.n} (D - d) = \frac{14322}{2.04 \times 6} (2.04 - 0.25) \text{ N-m} \]

= 2094.5 \text{ N-m} = 2094.5 \times 10^3 \text{ N-mm}

and section modulus for the cross-section of the arm,

\[ Z = \frac{\pi}{32} \times b_1 (a_1)^2 = \frac{\pi}{32} \times 0.5 a_1 (a_1)^2 = 0.05 (a_1)^3 \]
We know that the bending stress \( \sigma_b \),
\[
15 = \frac{M}{Z} = \frac{2094.5 \times 10^3}{0.05(a_1)^3} = \frac{41890 \times 10^3}{(a_1)^3}
\]
\[\therefore (a_1)^3 = 41890 \times 10^3 / 15 = 2793 \times 10^3 \text{ or } a_1 = 140 \text{ mm Ans.}\]
and
\[b_1 = 0.5 a_1 = 0.5 \times 140 = 70 \text{ mm Ans.}\]

6. **Dimensions of key**

The standard dimensions of rectangular sunk key for a shaft of diameter 125 mm are as follows:

- **Width of key**, \( w = 36 \text{ mm Ans.} \)
- **and thickness of key** \( = 20 \text{ mm Ans.} \)

The length of key \( (L) \) is obtained by considering the failure of key in shearing.

We know that the maximum torque transmitted by the shaft \( (T_{max}) \),
\[
14322 \times 10^3 = L \times w \times \tau \times \frac{d_1}{2} = L \times 36 \times 40 \times \frac{125}{2} = 90 \times 10^3 L
\]
\[\therefore L = 14322 \times 10^3 / 90 \times 10^3 = 159 \text{ say } 160 \text{ mm Ans.}\]

Let us now check the total stress in the rim which should not be greater than 15 MPa. We know that total stress in the rim,
\[
\sigma = \rho \omega^2 \left[ 0.75 + \frac{4.935 R}{n^2 \cdot \tau} \right]
\]
\[
= 7220 (26.8)^2 \left[ 0.75 + \frac{4.935 (2.04 / 2)}{6^2 \times 0.235} \right] \text{ N/m}^2
\]
\[
= 5.18 \times 10^6 (0.75 + 0.595) = 6.97 \times 10^6 \text{ N/m}^2 = 6.97 \text{ MPa}
\]
Since it is less than 15 MPa, therefore the design is safe.

**Example 22.11.** A single cylinder double acting steam engine delivers 185 kW at 100 r.p.m. The maximum fluctuation of energy per revolution is 15 per cent of the energy developed per revolution. The speed variation is limited to 1 per cent either way from the mean. The mean diameter of the rim is 2.4 m. Design and draw two views of the flywheel.

**Solution.** Given : \( P = 185 \text{ kW} = 185 \times 10^3 \text{ W} ; N = 100 \text{ r.p.m} ; \Delta E = 15\% ; E = 0.15 \text{ E} ; D = 2.4 \text{ m} \text{ or } R = 1.2 \text{ m} \)

1. **Mass of the flywheel rim**

Let
\( m = \text{Mass of the flywheel rim in kg.} \)

We know that the workdone or energy developed per revolution,
\[
E = \frac{P \times 60}{N} = \frac{185 \times 10^3 \times 60}{100} = 111000 \text{ N-m}
\]
\[\therefore \text{ Maximum fluctuation of energy,} \]
\[
\Delta E = 0.15 E = 0.15 \times 111000 = 16650 \text{ N-m}
\]

Since the speed variation is 1\% either way from the mean, therefore the total fluctuation of speed,
\[
N_1 - N_2 = 2\% \text{ of mean speed} = 0.02 N
\]
and coefficient of fluctuation of speed,
\[
C_s = \frac{N_1 - N_2}{N} = 0.02
\]
Velocity of the flywheel,
\[ v = \frac{\pi D \cdot N}{60} = \frac{\pi \times 2.4 \times 100}{60} = 12.57 \text{ m/s} \]
We know that the maximum fluctuation of energy (\(\Delta E\)),
\[ 16\,650 = m \cdot v^2 \cdot C_s = m \times (12.57)^2 \times 0.02 = 3.16 \text{ m} \]
\[ \therefore \quad m = \frac{16\,650}{3.16} = 5270 \text{ kg} \quad \text{Ans.} \]

2. **Cross-sectional dimensions of the flywheel rim**

Let \(t\) = Thickness of the flywheel rim in metres, and
\(b = \text{Width of the flywheel rim in metres} = 2t\) \(\text{(Assume)}\)

\[ \therefore \quad \text{Cross-sectional area of the rim,} \]
\[ A = b \times t = 2t \times t = 2t^2 \]
We know that mass of the flywheel rim \((m)\),
\[ 5270 = A \times \pi D \times \rho = 2t^2 \times \pi \times 2.4 \times 7200 = 108\,588 \, t^2 \]
\[ \text{(Taking } \rho = 7200 \text{ kg/m}^3) \]
\[ \therefore \quad t^2 = \frac{5270}{108\,588} = 0.0485 \quad \text{or} \quad t = 0.22 \text{ m} = 220 \text{ mm} \quad \text{Ans.} \]
and
\[ b = 2t = 2 \times 220 = 440 \text{ mm} \quad \text{Ans.} \]

3. **Diameter and length of hub**
Let \(d\) = Diameter of the hub,
\(d_1\) = Diameter of the shaft, and
\(l\) = Length of the hub,
We know that mean torque transmitted by the shaft,

\[ T_{\text{mean}} = \frac{P \times 60}{2 \pi N} = \frac{185 \times 10^3 \times 60}{2 \pi \times 100} = 17,664 \text{ N-m} \]

Assuming that the maximum torque transmitted \( T_{\text{max}} \) by the shaft is twice the mean torque, therefore

\[ T_{\text{max}} = 2 \times T_{\text{mean}} = 2 \times 17,664 = 35,328 \text{ N-m} = 35.328 \times 10^6 \text{ N-mm} \]

We also know that maximum torque transmitted by the shaft \( T_{\text{max}} \),

\[ 35.328 \times 10^6 = \frac{\pi}{16} \times \tau (d_i)^3 = \frac{\pi}{16} \times 40 (d_i)^3 = 7.855 (d_i)^3 \]

\[ \text{...(Assuming } \tau = 40 \text{ MPa } = 40 \text{ N/mm}^2) \]

\[ (d_i)^3 = 35.328 \times 10^6 / 7.855 = 4.5 \times 10^6 \text{ or } d_i = 165 \text{ mm Ans.} \]

The diameter of the hub \( d \) is made equal to twice the diameter of the shaft \( d_i \) and length of the hub \( l \) is equal to the width of the rim \( b \).

\[ d = 2d_i = 2 \times 165 = 330 \text{ mm} \text{; and } l = b = 440 \text{ mm Ans.} \]

4. **Cross-sectional dimensions of the elliptical arms**

Let \( a_1 = \) Major axis, 
\( b_1 = \) Minor axis = 0.5 \( a_1 \) ...(Assume)  
\( n = \) Number of arms = 6 ...(Assume)  
\( \sigma_B = \) Bending stress for the material of the arms  
\[ = 14 \text{ MPa } = 14 \text{ N/mm}^2 \] 

...(Assume)

We know that the maximum bending moment in the arm at the hub end which is assumed as cantilever is given by

\[ M = \frac{T}{R \cdot n} (R - r) = \frac{T}{D \cdot n} (D - d) = \frac{35,328}{2.4 \times 6} (2.4 - 0.33) \text{ N-m} \]

\[ = 5078 \text{ N-m} = 5078 \times 10^3 \text{ N-mm} \]

\[ \text{...(d is taken in metres)} \]

and section modulus for the cross-section of the arm,

\[ Z = \frac{\pi}{32} b_1 (a_1)^2 = \frac{\pi}{32} \times 0.5 a_1 (a_1)^2 = 0.05 (a_1)^3 \]

We know that the bending stress \( \sigma_B \),

\[ 14 = \frac{M}{Z} = \frac{5078 \times 10^3}{0.05 (a_1)^3} = \frac{101,560 \times 10^3}{(a_1)^3} \]

\[ \therefore (a_1)^3 = 101,560 \times 10^3 / 14 = 7254 \times 10^3 \]

or

\[ a_1 = 193.6 \text{ say } 200 \text{ mm Ans.} \]

and

\[ b_1 = 0.5 a_1 = 0.5 \times 200 = 100 \text{ mm Ans.} \]

5. **Dimensions of key**

The standard dimensions of rectangular sunk key for a shaft of 165 mm diameter are as follows:

Width of key, \( w = 45 \) mm Ans.

and thickness of key \( = 25 \) mm Ans.

The length of key \( L \) is obtained by considering the failure of key in shearing.

We know that the maximum torque transmitted by the shaft \( T_{\text{max}} \),

\[ 35.328 \times 10^6 = L \times w \times \tau \times \frac{d_i}{2} = L \times 45 \times 40 \times \frac{165}{2} = 148,500 L \]

\[ \therefore L = 35.328 \times 10^6 / 148,500 = 238 \text{ mm Ans.} \]
Let us now check the total stress in the rim which should not be greater than 14 MPa. We know that the total stress in the rim,

\[ \sigma = \rho \cdot v^2 \left( 0.75 + \frac{4.935 R}{n^2 \cdot t} \right) \]

\[ \sigma = 7200 \times (12.57)^2 \left( 0.75 + \frac{4.935 \times 1.2}{6^2 \times 0.22} \right) \text{ N/m}^2 \]

\[ = 1.14 \times 10^6 \times (0.75 + 0.75) = 1.71 \times 10^6 \text{ N/m}^2 = 1.71 \text{ MPa} \]

Since it is less than 14 MPa, therefore the design is safe.

**Example 22.12.** A punching press pierces 35 holes per minute in a plate using 10 kN-m of energy per hole during each revolution. Each piercing takes 40 per cent of the time needed to make one revolution. The punch receives power through a gear reduction unit which in turn is fed by a motor driven belt pulley 800 mm diameter and turning at 210 r.p.m. Find the power of the electric motor if overall efficiency of the transmission unit is 80 per cent. Design a cast iron flywheel to be used with the punching machine for a coefficient of steadiness of 5, if the space considerations limit the maximum diameter to 1.3 m.

**Allowable shear stress in the shaft material** = 50 MPa

**Allowable tensile stress for cast iron** = 4 MPa

**Density of cast iron** = 7200 kg/m^3

**Solution.**

Given:
- No. of holes = 35 per min
- Energy per hole = 10 kN-m = 10 000 N-m
- \( d = 800 \text{ mm} = 0.8 \text{ m} \)
- \( N = 210 \text{ r.p.m.} \)
- \( \eta = 80\% = 0.8 \)
- \( 1/C_s = 5 \) or \( C_s = 1/5 = 0.2 \)
- \( D_{max} = 1.3 \text{ m} \)
- \( \tau = 50 \text{ MPa} = 50 \text{ N/mm}^2 \)
- \( \sigma_t = 4 \text{ MPa} = 4 \text{ N/mm}^2 \)
- \( \rho = 7200 \text{ kg/m}^3 \)

**Power of the electric motor**

We know that energy used for piercing holes per minute

\[ = \text{No. of holes pierced} \times \text{Energy used per hole} \]

\[ = 35 \times 10 000 = 350 000 \text{ N-m/min} \]

\[ \therefore \text{Power needed for the electric motor,} \]

\[ P = \frac{\text{Energy used per minute}}{60 \times \eta} = \frac{350 000}{60 \times 0.8} = 7292 \text{ W} = 7.292 \text{ kW} \text{ Ans.} \]

**Design of cast iron flywheel**

First of all, let us find the maximum fluctuation of energy.

Since the overall efficiency of the transmission unit is 80%, therefore total energy to be supplied during each revolution,

\[ E_T = \frac{10 000}{0.8} = 12 500 \text{ N-m} \]

We know that velocity of the belt,

\[ v = \pi d \cdot N = \pi \times 0.8 \times 210 = 528 \text{ m/min} \]

\[ \therefore \text{Net tension or pull acting on the belt} \]

\[ = \frac{P \times 60}{v} = \frac{7292 \times 60}{528} = 828.6 \text{ N} \]

Since each piercing takes 40 per cent of the time needed to make one revolution, therefore time required to punch a hole

\[ = 0.4 / 35 = 0.0114 \text{ min} \]

and the distance moved by the belt during punching a hole

\[ = \text{Velocity of the belt} \times \text{Time required to punch a hole} \]

\[ = 528 \times 0.0114 = 6.03 \text{ m} \]
Energy supplied by the belt during punching a hole,
\[ E_B = \text{Net tension} \times \text{Distance travelled by belt} \]
\[ = 828.6 \times 6.03 = 4996 \text{ N-m} \]

Thus energy to be supplied by the flywheel for punching during each revolution or maximum fluctuation of energy,
\[ \Delta E = E_T - E_B = 12500 - 4996 = 7504 \text{ N-m} \]

1. **Mass of the flywheel**
   Let \( m \) = Mass of the flywheel rim.

   Since space considerations limit the maximum diameter of the flywheel as 1.3 m; therefore let us take the mean diameter of the flywheel,
   \[ D = 1.2 \text{ m or } R = 0.6 \text{ m} \]

   We know that angular velocity
   \[ \omega = \frac{2 \pi \times N}{60} = \frac{2 \pi \times 210}{60} = 22 \text{ rad / s} \]

   We also know that the maximum fluctuation of energy (\( \Delta E \)),
   \[ 7504 = m.R^2.\omega^2.C_s = m (0.6)^2 (22)^2 0.2 = 34.85 \text{ m} \]

   \[ m = \frac{7504}{34.85} = 215.3 \text{ kg Ans.} \]

2. **Cross-sectional dimensions of the flywheel rim**
   Let \( t \) = Thickness of the flywheel rim in metres, and \( b \) = Width of the flywheel rim in metres = 2 \( t \) \( ...(\text{Assume}) \)

   \[ A = b \times t = 2 t \times t = 2 t^2 \]

   We know that mass of the flywheel rim (\( m \)),
   \[ 215.3 = A \times \pi D \times \rho = 2 t^2 \times \pi \times 1.2 \times 7200 = 54.3 \times 10^3 t^2 \]

   \[ t^2 = \frac{215.3}{54.3 \times 10^3} = 0.00396 \]

   or \( t = 0.063 \text{ say 0.065 m} = 65 \text{ mm Ans.} \)

   and \( b = 2 t = 2 \times 65 = 130 \text{ mm Ans} \)

3. **Diameter and length of hub**
   Let \( d \) = Diameter of the hub,
   \( d_1 \) = Diameter of the shaft, and
   \( l \) = Length of the hub.

   First of all, let us find the diameter of the shaft (\( d_1 \)). We know that the mean torque transmitted by the shaft,

   \[ T_{\text{mean}} = \frac{P \times 60}{2 \pi N} = \frac{7292 \times 60}{2 \pi \times 210} = 331.5 \text{ N-m} \]

   Assuming that the maximum torque transmitted by the shaft is twice the mean torque, therefore maximum torque transmitted by the shaft (\( T_{\text{max}} \)),

   \[ T_{\text{max}} = 2 \times T_{\text{mean}} = 2 \times 331.5 = 663 \text{ N-m} = 663 \times 10^3 \text{ N-mm} \]

   We know that maximum torque transmitted by the shaft (\( T_{\text{max}} \)),

   \[ 663 \times 10^3 = \frac{\pi}{16} \times \tau (d_1)^3 = \frac{\pi}{16} \times 50 (d_1)^3 = 9.82 (d_1)^3 \]

   \[ (d_1)^3 = \frac{663 \times 10^3}{9.82} = 67.5 \times 10^3 \]

   \[ or \quad d_1 = 40.7 \text{ say 45 mm Ans.} \]
A Textbook of Machine Design

The diameter of the hub \((d)\) is made equal to twice the diameter of the shaft \((d_1)\) and length of hub \((l)\) is equal to the width of the rim \((b)\).

\[
\therefore \quad d = 2d_1 = 2 \times 45 = 90 \text{ mm} = 0.09 \text{ m} \quad \text{and} \quad l = b = 130 \text{ mm} \quad \text{Ans.}
\]

4. Cross-sectional dimensions of the elliptical cast iron arms

Let

- \(a_1 =\) Major axis,
- \(b_1 =\) Minor axis \(= 0.5 \ a_1\) \(\text{(Assume)}\)
- \(n =\) Number of arms \(= 6\) \(\text{(Assume)}\)

We know that the maximum bending moment in the arm at the hub end, which is assumed as cantilever is given by

\[
M = \frac{T}{R \cdot n} \quad (R - r) = \frac{T}{D \cdot n} \quad (D - d) = \frac{663 \times 1.2}{1.2 \times 6} \quad (1.2 - 0.09) \text{ N-m}
\]

\[
= 102.2 \text{ N-m} = 102 \text{ 200 N-mm}
\]

and section modulus for the cross-section of the arms,

\[
Z = \frac{\pi}{32} \times b_1 (a_1)^2 = \frac{\pi}{32} \times 0.5 a_1 (a_1)^2 = 0.05 (a_1)^3
\]

We know that bending stress \((\sigma)_t\),

\[
\frac{M}{Z} = \frac{102 \ 200}{0.05 (a_1)^3} = \frac{2044 \times 10^3}{(a_1)^3}
\]

\[
\therefore \quad (a_1)^3 = 2044 \times 10^3 / 4 = 511 \times 10^3 \quad \text{or} \quad a_1 = 80 \text{ mm} \quad \text{Ans.}
\]

and

\[
b_1 = 0.5 \ a_1 = 0.5 \times 80 = 40 \text{ mm} \quad \text{Ans.}
\]

5. Dimensions of key

The standard dimensions of rectangular sunk key for a shaft of diameter 45 mm are as follows:

- Width of key, \(w = 16 \text{ mm} \quad \text{Ans.}\)
- and thickness of key \(= 10 \text{ mm} \quad \text{Ans.}\)

The length of key \((L)\) is obtained by considering the failure of key in shearing.

We know that maximum torque transmitted by the shaft \((T_{\text{max}})\),

\[
663 \times 10^3 = L \times w \times \tau \times \frac{d_1}{2} = L \times 16 \times 50 \times \frac{45}{2} = 18 \times 10^3 L
\]

\[
\therefore \quad L = 663 \times 10^3/18 \times 10^3 = 36.8 \text{ say 38 mm} \quad \text{Ans.}
\]

Let us now check the total stress in the rim which should not be greater than 4 MPa.

We know that the velocity of the rim,

\[
v = \frac{\pi D \times N}{60} = \frac{\pi \times 1.2 \times 210}{60} = 13.2 \text{ m/s}
\]

\[
\therefore \quad \text{Total stress in the rim,}
\]

\[
\sigma = \rho \cdot v^2 \left( 0.75 + \frac{4.935 \ R}{n^2 \cdot I} \right) = 7200 \times (13.2)^2 \left[ 0.75 + \frac{4.935 \times 0.6}{6^2 \times 0.065} \right]
\]

\[
= 1.25 \times 10^6 \ (0.75 + 1.26) = 2.5 \times 10^6 \text{ N/m}^2 = 2.5 \text{ MPa}
\]

Since it is less than 4 MPa, therefore the design is safe.

22.11 Construction of Flywheels

The flywheels of smaller size (upto 600 mm diameter) are casted in one piece. The rim and hub are joined together by means of web as shown in Fig. 22.19 \(a\). The holes in the web may be made for handling purposes.
In case the flywheel is of larger size (upto 2.5 metre diameter), the arms are made instead of web, as shown in Fig. 22.19 (b). The number of arms depends upon the size of flywheel and its speed of rotation. But the flywheels above 2.5 metre diameter are usually casted in two piece. Such a flywheel is known as split flywheel. A split flywheel has the advantage of relieving the shrinkage stresses in the arms due to unequal rate of cooling of casting. A flywheel made in two halves should be spilt at the arms rather than between the arms, in order to obtain better strength of the joint. The two halves of the flywheel are connected by means of bolts through the hub, as shown in Fig. 22.20. The two halves are also joined at the rim by means of cotter joint (as shown in Fig. 22.20) or shrink links (as shown in Fig. 22.21). The width or depth of the shrink link is taken as 1.25 to 1.35 times the thickness of link. The slot in the rim into which the link is inserted is made slightly larger than the size of link.

(a) Flywheel with web.  (b) Flywheel with arms.

Fig. 22.19

Fig. 22.20. Split flywheel.
The relative strength of a rim joint and the solid rim are given in the following table.

**Table 22.3 Relative strength of a rim joint and the solid rim.**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Type of construction</th>
<th>Relative strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Solid rim.</td>
<td>1.00</td>
</tr>
<tr>
<td>2.</td>
<td>Flanged joint, bolted, rim parted between arms.</td>
<td>0.25</td>
</tr>
<tr>
<td>3.</td>
<td>Flanged joint, bolted, rim parted on an arm.</td>
<td>0.50</td>
</tr>
<tr>
<td>4.</td>
<td>Shrink link joint.</td>
<td>0.60</td>
</tr>
<tr>
<td>5.</td>
<td>Cotter or anchor joints.</td>
<td>0.70</td>
</tr>
</tbody>
</table>

**Example 22.13.** A split type flywheel has outside diameter of the rim 1.80 m, inside diameter 1.35 m and the width 300 mm, the two halves of the wheel are connected by four bolts through the hub and near the rim joining the split arms and also by four shrink links on the rim. The speed is 250 r.p.m. and a turning moment of 15 kN-m is to be transmitted by the rim. Determine:

1. The diameter of the bolts at the hub and near the rim, $\sigma_{tb} = 35$ MPa.
2. The cross-sectional dimensions of the rectangular shrink links at the rim, $\sigma_{tl} = 40$ MPa; $w = 1.25 h$.
3. The cross-sectional dimensions of the elliptical arms at the hub and rim if the wheel has six arms, $\sigma_{ta} = 15$ MPa, minor axis being 0.5 times the major axis and the diameter of shaft being 150 mm.

Assume density of the material of the flywheel as 7200 kg/m$^3$.

**Solution.** Given: $D_0 = 1.8$ m; $D_i = 1.35$ m; $b = 300$ mm = 0.3 m; $N = 250$ r.p.m.; $T = 15$ kN-m = 15 000 N-m; $\sigma_{tb} = 35$ MPa = 35 N/mm$^2$; $\sigma_{tl} = 40$ MPa = 40 N/mm$^2$; $w = 1.25 h$; $n = 6$; $b_1 = 0.5 a_1$; $\sigma_{ta} = 15$ MPa = 15 N/mm$^2$; $d_1 = 150$ mm; $\rho = 7200$ kg/m$^3$.

1. **Diameter of the bolts at the hub and near the rim**

   Let $d_c$ = Core diameter of the bolts in mm.
   
   We know that mean diameter of the rim, 
   
   \[ D = \frac{D_0 + D_i}{2} = \frac{1.8 + 1.35}{2} = 1.575 \text{ m} \]
   
   and thickness of the rim, 
   
   \[ t = \frac{D_0 - D_i}{2} = \frac{1.8 - 1.35}{2} = 0.225 \text{ m} \]

   Peripheral speed of the flywheel, 
   
   \[ v = \frac{\pi D \cdot N}{60} = \frac{\pi \times 1.575 \times 250}{60} = 20.6 \text{ m/s} \]

   We know that centrifugal stress (or tensile stress) at the rim, 
   
   \[ \sigma_t = \rho \times v^2 = 7200 (20.6)^2 = 3.1 \times 10^6 \text{ N/m}^2 = 3.1 \text{ N/mm}^2 \]

   Cross-sectional area of the rim, 
   
   \[ A = b \times t = 0.3 \times 0.225 = 0.0675 \text{ m}^2 \]

   \[ \therefore \text{Maximum tensile force acting on the rim} \]
   
   \[ = \sigma_t \times A = 3.1 \times 10^6 \times 0.0675 = 209250 \text{ N} \]
We know that tensile strength of the four bolts
\[ \sigma_{tb} = \frac{\pi (d_c)^2}{4 \times \text{No. of bolts}} \times 35 \times 4 = 110 (d_c)^2 \quad \text{(ii)} \]
Since the bolts are made as strong as the rim joint, therefore from equations (i) and (ii), we have
\[ (d_c)^2 = 209250 / 110 = 1903 \text{ or } d_c = 43.6 \text{ mm} \]
The standard size of the bolt is M 56 with \( d_c = 48.65 \text{ mm} \) Ans.

2. Cross-sectional dimensions of rectangular shrink links at the rim
Let
\[ h = \text{Depth of the link in mm, and} \]
\[ w = \text{Width of the link in mm} = 1.25 h \quad \text{(Given)} \]
\[ \therefore \text{Cross-sectional area of each link,} \]
\[ A_l = w \times h = 1.25 h^2 \text{ mm}^2 \]
We know that the maximum tensile force on half the rim
\[ = 2 \times \sigma_{r} \text{ for rim} \times \text{Cross-sectional area of rim} \]
\[ = 2 \times 3.1 \times 10^6 \times 0.0675 = 418500 \text{ N} \quad \text{(iii)} \]
and tensile strength of the four shrink links
\[ = \sigma_{tl} \times A_l = 40 \times 1.25 h^2 \times 4 = 200 h^2 \quad \text{(iv)} \]
From equations (iii) and (iv), we have
\[ h^2 = 418500 \times 200 = 209250 \text{ or } h = 45.7 \text{ say } 46 \text{ mm} \text{ Ans.} \]
and
\[ w = 1.25 h = 1.25 \times 46 = 57.5 \text{ say } 58 \text{ mm} \text{ Ans.} \]

3. Cross-sectional dimensions of the elliptical arms
Let
\[ a_1 = \text{Major axis}, \]
\[ b_1 = \text{Minor axis} = 0.5 a_1 \quad \text{(Given)} \]
\[ n = \text{Number of arms} = 6 \quad \text{(Given)} \]
Since the diameter of shaft \( d_1 \) is 150 mm and the diameter of hub \( d \) is taken equal to twice the diameter of shaft, therefore
\[ d = 2 d_1 = 2 \times 150 = 300 \text{ mm} = 0.3 \text{ m} \]
We know that maximum bending moment on arms at the hub end,
\[ M = \frac{T}{R \times n} (R - r) = \frac{T}{D \times n} (D - d) = \frac{15000}{1.575 \times 6} (1.575 - 0.3) \]
\[ = 2024 \text{ N-m} = 2024 \times 10^3 \text{ N-mm} \]
Section modulus,
\[ Z = \frac{\pi}{32} \times b_1 (a_1)^2 = \frac{\pi}{32} \times 0.5 a_1 (a_1)^2 = 0.05 (a_1)^3 \]
We know that bending stress for arms \( (\sigma_{a1}) \),
\[ 15 = \frac{M}{Z} = \frac{2024 \times 10^3}{0.05 (a_1)^3} = \frac{40.5 \times 10^6}{(a_1)^3} \]
\[ \therefore (a_1)^3 = 40.5 \times 10^6 / 15 = 2.7 \times 10^6 \text{ or } a_1 = 139.3 \text{ say } 140 \text{ mm} \text{ Ans.} \]
and
\[ b_1 = 0.5 a_1 = 0.5 \times 140 = 70 \text{ mm} \text{ Ans.} \]

EXERCISES

1. The turning moment diagram for a multicylinder engine has been drawn to a scale of 1 mm = 1000 N-m and 1 mm = 6º. The areas above and below the mean turning moment line taken in order are 530, 380, 470, 180, 350 and 280 sq.mm.
For the engine, find the diameter of the flywheel. The mean r.p.m is 150 and the total fluctuation of speed must not exceed 3.5% of the mean.
Determine a suitable cross-sectional area of the rim of the flywheel, assuming the total energy of the flywheel to be \( \frac{15}{14} \) that of the rim. The peripheral velocity of the flywheel is 15 m/s.

2. A machine has to carry out punching operation at the rate of 10 holes/min. It does 6 N-m of work per sq mm of the sheared area in cutting 25 mm diameter holes in 20 mm thick plates. A flywheel is fitted to the machine shaft which is driven by a constant torque. The fluctuation of speed is between 180 and 200 r.p.m. Actual punching takes 1.5 seconds. Frictional losses are equivalent to 1/6 of the workdone during punching. Find:

(a) Power required to drive the punching machine, and
(b) Mass of the flywheel, if radius of gyration of the wheel is 450 mm.

3. The turning moment diagram for an engine is drawn to the following scales:

\[ 1 \text{ mm} = 3100 \text{ N-m} ; 1 \text{ mm} = 1.6^\circ \]

The areas of the loops above and below the mean torque line taken in order are: 77, 219, 588, 522, 97, 116, 1200 and 1105 mm².

The mean speed of the engine is 300 r.p.m. and the permissible fluctuation in speed is ± 2 per cent of mean speed. The stress in the material of the rim is not to exceed 4.9 MPa and density of its material is 7200 kg/m³. Assuming that the rim stores \( \frac{15}{16} \) of the energy that is stored by the flywheel, estimate

(a) Diameter of rim; and
(b) Area of cross-section of rim.

4. A single cylinder internal combustion engine working on the four stroke cycle develops 75 kW at 360 r.p.m. The fluctuation of energy can be assumed to be 0.9 times the energy developed per cycle. If the fluctuation of speed is not to exceed 1 per cent and the maximum centrifugal stress in the flywheel is to be 5.5 MPa, estimate the mean diameter and the cross-sectional area of the rim. The material of the rim has a density of 7200 kg/m³.

[Ans. 1.464 m ; 0.09 m²]

5. Design a cast iron flywheel for a four stroke cycle engine to develop 110 kW at 150 r.p.m. The work done in the power stroke is 1.3 times the average work done during the whole cycle. Take the mean diameter of the flywheel as 3 metres. The total fluctuation of speed is limited to 5 per cent of the mean speed. The material density is 7250 kg/m³. The permissible shear stress for the shaft material is 40 MPa and flexural stress for the arms of the flywheel is 20 MPa.

6. A punching press is required to punch 40 mm diameter holes in a plate of 15 mm thickness at the rate of 30 holes per minute. It requires 6 N-m of energy per mm² of sheared area. Determine the moment of inertia of the flywheel if the punching takes one-tenth of a second and the r.p.m. of the flywheel varies from 160 to 140.

7. A punch press is fitted with a flywheel capable of furnishing 3000 N-m of energy during quarter of a revolution near the bottom dead centre while blanking a hole on sheet metal. The maximum speed of the flywheel during the operation is 200 r.p.m. and the speed decreases by 10% during the cutting stroke. The mean radius of the rim is 900 mm. Calculate the approximate mass of the flywheel rim assuming that it contributes 90% of the energy requirements.

8. A punching machine makes 24 working strokes per minute and is capable of punching 30 mm diameter holes in 20 mm thick steel plates having an ultimate shear strength of 350 MPa. The punching operation takes place during \( \frac{1}{10} \) th of a revolution of the crankshaft. Find the power required for the driving motor, assuming a mechanical efficiency of 76%. Determine suitable dimensions for the rim cross-section of the flywheel, which revolves at 9 times the speed of crankshaft. The permissible coefficient of fluctuation of speed is 0.4.

The flywheel is to be made of cast iron having a safe tensile stress of 6 MPa and density 7250 kg/m³. The diameter of the flywheel must not exceed 1.05 m owing to space restrictions. The hub and spokes
may be assumed to provide 5% of the rotational inertia of the wheel. Check for the centrifugal stress induced in the rim.

9. Design completely the flywheel, shaft and the key for securing the flywheel to the shaft, for a punching machine having a capacity of producing 30 holes of 20 mm diameter per minute in steel plate 16 mm thickness. The ultimate shear stress for the material of the plate is 360 MPa. The actual punching operation estimated to last for a period of 36º rotation of the punching machine crankshaft. This crank shaft is powered by a flywheel shaft through a reduction gearing having a ratio 1 : 8. Assume that the mechanical efficiency of the punching machine is 80% and during the actual punching operation the flywheel speed is reduced by a maximum of 10%. The diameter of flywheel is restricted to 0.75 m due to space limitations.

10. A cast iron wheel of mean diameter 3 metre has six arms of elliptical section. The energy to be stored in it is 560 kN-m when rotating at 120 r.p.m. The speed of the mean diameter is 18 m/s. Calculate the following:
   (a) Assuming that the whole energy is stored in the rim, find the cross-section, if the width is 300 mm.
   (b) Find the cross-section of the arms near the boss on the assumption that their resistance to bending is equal to the torsional resistance of the shaft which is 130 mm in diameter.

The maximum shear stress in the shaft is to be within 63 MPa and the tensile stress 16 MPa. Assume the minor axis of the ellipse to be 0.65 major axis.

11. A cast iron flywheel is to be designed for a single cylinder double acting steam engine which delivers 150 kW at 80 r.p.m. The maximum fluctuation of energy per revolution is 10%. The total fluctuation of the speed is 4 per cent of the mean speed. If the mean diameter of the flywheel rim is 2.4 metres, determine the following:
   (a) Cross-sectional dimensions of the rim, assuming that the hub and spokes provide 5% of the rotational inertia of the wheel. The density of cast iron is 7200 kg/m³ and tensile stress 16 MPa.
   Take width of rim equal to twice of thickness.
   (b) Dimensions of hub and rectangular sunk key. The shear stress for the material of shaft and key is 40 MPa.
   (c) Cross-sectional dimensions of the elliptical arms assuming major axis as twice of minor axis and number of arms equal to six.

12. Design a cast iron flywheel having six arms for a four stroke engine developing 120 kW at 150 r.p.m. The mean diameter of the flywheel may be taken as 3 metres. The fluctuation of speed is 2.5% of mean speed. The workdone during the working stroke is 1.3 times the average workdone during the whole cycle. Assume allowable shear stress for the shaft and key as 40 MPa and tensile stress for cast iron as 20 MPa. The following proportions for the rim and elliptical arms may be taken:
   (a) Width of rim = 2 × Thickness of rim
   (b) Major axis = 2 × Minor axis.

13. A multi-cylinder engine is to run at a speed of 500 r.p.m. On drawing the crank effort diagram to scale 1 mm = 2500 N-m and 1 mm = 3°, the areas above and below the mean torque line are in sq mm as below:
   + 160, – 172, + 168, – 191, + 197, – 162

The speed is to be kept within ± 1% of the mean speed of the engine. Design a suitable rim type C.I. flywheel for the above engine. Assume rim width as twice the thickness and the overhang of the flywheel from the centre of the nearest bearing as 1.2 metres. The permissible stresses for the rim in tension is 6 MPa and those for shaft and key in shear are 42 MPa. The allowable stress for the arm is 14 MPa. Sketch a dimensioned end view of the flywheel.
14. An engine runs at a constant load at a speed of 480 r.p.m. The crank effort diagram is drawn to a scale
1 mm = 200 N-m torque and 1 mm = 3.6° crank angle. The areas of the diagram above and below the
mean torque line in sq mm are in the following order:
+ 110, – 132, + 153, – 166, + 197, – 162

Design the flywheel if the total fluctuation of speed is not to exceed 10 r.p.m. and the centrifugal stress
in the rim is not to exceed 5 MPa. You may assume that the rim breadth is approximately 2.5 times the
rim thickness and 90% of the moment of inertia is due to the rim. The density of the material of the
flywheel is 7250 kg/m³.

Make a sketch of the flywheel giving the dimensions of the rim, the mean diameter of the rim and
other estimated dimensions of spokes, hub etc.

15. A four stroke oil engine developing 75 kW at 300 r.p.m is to have the total fluctuation of speed limited
to 5%. Two identical flywheels are to be designed. The workdone during the power stroke is found to
be 1.3 times the average workdone during the whole cycle. The turning moment diagram can be
approximated as a triangle during the power stroke. Assume that the hoop stress in the flywheel and
the bending stress in the arms should not exceed 25 MPa. The shear stress in the key and shaft
material should not exceed 40 MPa. Give a complete design of the flywheel. Assume four arms of
elliptical cross-section with the ratio of axes 1 : 2. Design should necessarily include (i) moment of
inertia of the flywheel, (ii) flywheel rim dimensions, (iii) arm dimensions, and (iv) flywheel boss and
key dimensions and sketch showing two views of the flywheel with all the dimensions.

QUESTIONS

1. What is the main function of a flywheel in an engine?
2. In what way does a flywheel differ from that of a governor? Illustrate your answer with suitable
examples.
3. Explain why flywheels are used in punching machines. Does the mounting of a flywheel reduce the
stress induced in the shafts.
4. Define ‘coefficient of fluctuation of speed’ and ‘coefficient of steadiness’.
5. What do you understand by ‘fluctuation of energy’ and ‘maximum fluctuation of energy’.
6. Define ‘coefficient of fluctuation of energy’.
7. Discuss the various types of stresses induced in a flywheel rim.
8. Explain the procedure for determining the size and mass of a flywheel with the help of a turning
moment diagram.
9. Discuss the procedure for determining the cross-sectional dimensions of arms of a flywheel.
10. State the construction of flywheels.

OBJECTIVE TYPE QUESTIONS

1. The maximum fluctuation of speed is the
   (a) difference of minimum fluctuation of speed and the mean speed
   (b) difference of the maximum and minimum speeds
   (c) sum of the maximum and minimum speeds
   (d) variations of speed above and below the mean resisting torque line

2. The coefficient of fluctuation of speed is the ........... of maximum fluctuation of speed and the mean
speed.
   (a) product
   (b) ratio
   (c) sum
   (d) difference
3. In a turning moment diagram, the variations of energy above and below the mean resisting torque line is called
   (a) fluctuation of energy  (b) maximum fluctuation of energy
   (c) coefficient of fluctuation of energy  (d) none of these

4. If \( E = \) Mean kinetic energy of the flywheel, \( C_S = \) Coefficient of fluctuation of speed and \( \Delta E = \) Maximum fluctuation of energy, then
   (a) \( \Delta E = \frac{E}{C_S} \)  (b) \( \Delta E = E^2 \times C_S \)
   (c) \( \Delta E = E \times C_S \)  (d) \( \Delta E = 2E \times C_S \)

5. The ratio of the maximum fluctuation of energy to the ....... is called coefficient of fluctuation of energy.
   (a) minimum fluctuation of energy  (b) work done per cycle

6. Due to the centrifugal force acting on the rim, the flywheel arms will be subjected to
   (a) tensile stress  (b) compressive stress
   (c) shear stress  (d) none of these

7. The tensile stress in the flywheel rim due to the centrifugal force acting on the rim is given by
   (a) \( \frac{\rho v^2}{4} \)  (b) \( \frac{\rho v^2}{2} \)
   (c) \( \frac{3\rho v^2}{4} \)  (d) \( \rho v^2 \)
   where \( \rho = \) Density of the flywheel material, and \( v = \) Linear velocity of the flywheel.

8. The cross-section of the flywheel arms is usually
   (a) elliptical  (b) rectangular
   (c) I-section  (d) L-section

9. In order to find the maximum bending moment on the arms, it is assumed as a
   (a) simply supported beam carrying a uniformly distributed load over the arm
   (b) fixed at both ends (i.e. at the hub and at the free end of the rim) and carrying a uniformly distributed load over the arm.
   (c) cantilever beam fixed at the hub and carrying a concentrated load at the free end of the rim
   (d) none of the above

10. The diameter of the hub of the flywheel is usually taken
    (a) equal to the diameter of the shaft  (b) twice the diameter of the shaft
     (c) three times the diameter of the shaft  (d) four times the diameter of the shaft

### ANSWERS

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(b)</td>
<td>2.</td>
<td>(b)</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>(d)</td>
<td>5.</td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>(a)</td>
<td>7.</td>
<td>(d)</td>
<td>8.</td>
</tr>
<tr>
<td>9.</td>
<td>(c)</td>
<td>10.</td>
<td>(b)</td>
<td></td>
</tr>
</tbody>
</table>
32 Internal Combustion Engine Parts

1. Introduction.
2. Principal Parts of an I. C. Engine.
5. Piston.
6. Design Considerations for a Piston.
7. Material for Pistons.
8. Piston Head or Crown.
11. Piston skirt.
12. Piston Pin.
13. Connecting Rod.
14. Forces Acting on the Connecting Rod.
15. Design of Connecting Rod.
17. Material and Manufacture of Crankshafts.
18. Bearing Pressures and Stresses in Crankshafts.
20. Design for Centre Crankshaft.
21. Side or Overhung Crankshaft.
22. Valve Gear Mechanism.
23. Valves.
24. Rocker Arm.

32.1 Introduction

As the name implies, the internal combustion engines (briefly written as I. C. engines) are those engines in which the combustion of fuel takes place inside the engine cylinder. The I.C. engines use either petrol or diesel as their fuel. In petrol engines (also called spark ignition engines or S.I engines), the correct proportion of air and petrol is mixed in the carburettor and fed to engine cylinder where it is ignited by means of a spark produced at the spark plug. In diesel engines (also called compression ignition engines or C.I engines), only air is supplied to the engine cylinder during suction stroke and it is compressed to a very high pressure, thereby raising its temperature from 600°C to 1000°C. The desired quantity of fuel (diesel) is now injected into the engine cylinder in the form of a very fine spray and gets ignited when comes in contact with the hot air.

The operating cycle of an I.C. engine may be completed either by the two strokes or four strokes of the
piston. Thus, an engine which requires two strokes of the piston or one complete revolution of the crankshaft to complete the cycle, is known as **two stroke engine**. An engine which requires four strokes of the piston or two complete revolutions of the crankshaft to complete the cycle, is known as **four stroke engine**.

The two stroke petrol engines are generally employed in very light vehicles such as scooters, motor cycles and three wheelers. The two stroke diesel engines are generally employed in marine propulsion.

The four stroke petrol engines are generally employed in light vehicles such as cars, jeeps and also in aeroplanes. The four stroke diesel engines are generally employed in heavy duty vehicles such as buses, trucks, tractors, diesel locomotive and in the earth moving machinery.

### 32.2 Principal Parts of an Engine

The principal parts of an I.C engine, as shown in Fig. 32.1 are as follows:


The design of the above mentioned principal parts are discussed, in detail, in the following pages.

![Fig. 32.1. Internal combustion engine parts.](image)

### 32.3 Cylinder and Cylinder Liner

The function of a cylinder is to retain the working fluid and to guide the piston. The cylinders are usually made of cast iron or cast steel. Since the cylinder has to withstand high temperature due to the combustion of fuel, therefore, some arrangement must be provided to cool the cylinder. The single cylinder engines (such as scooters and motorcycles) are generally air cooled. They are provided with fins around the cylinder. The multi-cylinder engines (such as of cars) are provided with water jackets around the cylinders to cool it. In smaller engines, the cylinder, water jacket and the frame are
made as one piece, but for all the larger engines, these parts are manufactured separately. The cylinders are provided with cylinder liners so that in case of wear, they can be easily replaced. The cylinder liners are of the following two types:

1. Dry liner, and 2. Wet liner.

![Fig. 32.2. Dry and wet liner.](image)

A cylinder liner which does not have any direct contact with the engine cooling water, is known as **dry liner**, as shown in Fig. 32.2 (a). A cylinder liner which has its outer surface in direct contact with the engine cooling water, is known as **wet liner**, as shown in Fig. 32.2 (b).

The cylinder liners are made from good quality close grained cast iron (i.e. pearlitic cast iron), nickel cast iron, nickel chromium cast iron. In some cases, nickel chromium cast steel with molybdenum may be used. The inner surface of the liner should be properly heat-treated in order to obtain a hard surface to reduce wear.

### 32.4 Design of a Cylinder

In designing a cylinder for an I. C. engine, it is required to determine the following values:

1. **Thickness of the cylinder wall.** The cylinder wall is subjected to gas pressure and the piston side thrust. The gas pressure produces the following two types of stresses:
   
   (a) Longitudinal stress, and (b) Circumferential stress.

 suitability
Since these two stresses act at right angles to each other, therefore, the net stress in each direction is reduced.

The piston side thrust tends to bend the cylinder wall, but the stress in the wall due to side thrust is very small and hence it may be neglected.

Let

- \(D_0\) = Outside diameter of the cylinder in mm,
- \(D\) = Inside diameter of the cylinder in mm,
- \(p\) = Maximum pressure inside the engine cylinder in N/mm\(^2\),
- \(t\) = Thickness of the cylinder wall in mm, and
- \(1/m\) = Poisson’s ratio. It is usually taken as 0.25.

The apparent longitudinal stress is given by

\[
\sigma_l = \frac{\text{Force}}{\text{Area}} = \frac{\pi \times D^2 \times p}{\frac{\pi}{4} [(D_0)^2 - D^2]} = \frac{D^2 \times p}{(D_0)^2 - D^2}
\]

and the apparent circumferential stress is given by

\[
\sigma_c = \frac{\text{Force}}{\text{Area}} = \frac{D \times l \times p}{2 \times r \times l} = \frac{D \times p}{2r}
\]

... (where \(l\) is the length of the cylinder and area is the projected area)

\[\therefore\] Net longitudinal stress = \(\sigma_l - \frac{\sigma_c}{m}\)

and net circumferential stress = \(\sigma_c - \frac{\sigma_l}{m}\)

The thickness of a cylinder wall \((t)\) is usually obtained by using a thin cylindrical formula, \(i.e.,\)

\[
t = \frac{p \times D}{2 \sigma_c} + C
\]

where

- \(p\) = Maximum pressure inside the cylinder in N/mm\(^2\),
- \(D\) = Inside diameter of the cylinder or cylinder bore in mm,
- \(\sigma_c\) = Permissible circumferential or hoop stress for the cylinder material in MPa or N/mm\(^2\). Its value may be taken from 35 MPa to 100 MPa depending upon the size and material of the cylinder.
- \(C\) = Allowance for reboring.

The allowance for reboring \((C)\) depending upon the cylinder bore \((D)\) for I. C. engines is given in the following table:

<table>
<thead>
<tr>
<th>(D) (mm)</th>
<th>75</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
<th>350</th>
<th>400</th>
<th>450</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C) (mm)</td>
<td>1.5</td>
<td>2.4</td>
<td>4.0</td>
<td>6.3</td>
<td>8.0</td>
<td>9.5</td>
<td>11.0</td>
<td>12.5</td>
<td>12.5</td>
<td>12.5</td>
</tr>
</tbody>
</table>

The thickness of the cylinder wall usually varies from 4.5 mm to 25 mm or more depending upon the size of the cylinder. The thickness of the cylinder wall \((t)\) may also be obtained from the following empirical relation, \(i.e.,\)

\[t = 0.045 \times D + 1.6\text{ mm}\]

The other empirical relations are as follows:

Thickness of the dry liner

\[= 0.03 \times D\text{ to } 0.035 \times D\]
Thickness of the water jacket wall

\[ D + 1.6 \text{ mm or } t/3 \text{ mm for bigger cylinders and } 3t/4 \text{ for smaller cylinders} \]

Water space between the outer cylinder wall and inner jacket wall

\[ 10 \text{ mm for a 75 mm cylinder to 75 mm for a 750 mm cylinder} \]

or \[ 0.08D + 6.5 \text{ mm} \]

2. **Bore and length of the cylinder.** The bore (i.e. inner diameter) and length of the cylinder may be determined as discussed below:

Let

\[ p_m = \text{Indicated mean effective pressure in N/mm}^2, \]
\[ D = \text{Cylinder bore in mm}, \]
\[ A = \text{Cross-sectional area of the cylinder in mm}^2, \]
\[ = \pi D^2/4 \]
\[ l = \text{Length of stroke in metres}, \]
\[ N = \text{Speed of the engine in r.p.m., and} \]
\[ n = \text{Number of working strokes per min} \]
\[ = N, \text{for two stroke engine} \]
\[ = N/2, \text{for four stroke engine.} \]

We know that the power produced inside the engine cylinder, i.e. indicated power,

\[ I.P. = \frac{p_m \times l \times A \times n}{60} \text{ watts} \]

From this expression, the bore \((D)\) and length of stroke \((l)\) is determined. The length of stroke is generally taken as 1.25 \(D\) to 2\(D\).

Since there is a clearance on both sides of the cylinder, therefore length of the cylinder is taken as 15 percent greater than the length of stroke. In other words,

Length of the cylinder, \(L = 1.15 \times \text{Length of stroke} = 1.15l \)

**Notes:**

(a) If the power developed at the crankshaft, i.e. brake power \((B.P.)\) and the mechanical efficiency \((\eta_m)\) of the engine is known, then

\[ I.P. = \frac{B.P.}{\eta_m} \]

(b) The maximum gas pressure \((p)\) may be taken as 9 to 10 times the mean effective pressure \((p_m)\).

3. **Cylinder flange and studs.** The cylinders are cast integral with the upper half of the crankcase or they are attached to the crankcase by means of a flange with studs or bolts and nuts. The cylinder flange is integral with the cylinder and should be made thicker than the cylinder wall. The flange thickness should be taken as 1.2 \(t\) to 1.4 \(t\), where \(t\) is the thickness of cylinder wall.

The diameter of the studs or bolts may be obtained by equating the gas load due to the maximum pressure in the cylinder to the resisting force offered by all the studs or bolts. Mathematically,

\[ \frac{\pi}{4} \times D^2 \times p = n_s \times \frac{\pi}{4} \left(d_c\right)^2 \sigma, \]

where

\[ D = \text{Cylinder bore in mm,} \]
\[ p = \text{Maximum pressure in N/mm}^2, \]
\[ n_s = \text{Number of studs. It may be taken as 0.01 \(D + 4\) to 0.02 \(D + 4\)} \]
\[ d_c = \text{Core or minor diameter, i.e. diameter at the root of the thread in mm,} \]
\( \sigma_t = \) Allowable tensile stress for the material of studs or bolts in MPa or N/mm\(^2\). It may be taken as 35 to 70 MPa.

The nominal or major diameter of the stud or bolt (\(d\)) usually lies between 0.75 \(t_f\) to \(t_f\), where \(t_f\) is the thickness of flange. In no case, a stud or bolt less than 16 mm diameter should be used.

The distance of the flange from the centre of the hole for the stud or bolt should not be less than \(d + 6\) mm and not more than 1.5 \(d\), where \(d\) is the nominal diameter of the stud or bolt.

In order to make a leak-proof joint, the pitch of the studs or bolts should lie between 19\(\sqrt{d}\) to 28.5\(\sqrt{d}\), where \(d\) is in mm.

4. Cylinder head. Usually, a separate cylinder head or cover is provided with most of the engines. It is, usually, made of box type section of considerable depth to accommodate ports for air and gas passages, inlet valve, exhaust valve and spark plug (in case of petrol engines) or atomiser at the centre of the cover (in case of diesel engines).

The cylinder head may be approximately taken as a flat circular plate whose thickness (\(t_h\)) may be determined from the following relation:

\[
t_h = D \sqrt{\frac{C \cdot p}{\sigma_c}}
\]

where

\(D\) = Cylinder bore in mm,

\(p\) = Maximum pressure inside the cylinder in N/mm\(^2\),

\(\sigma_c\) = Allowable circumferential stress in MPa or N/mm\(^2\). It may be taken as 30 to 50 MPa, and

\(C\) = Constant whose value is taken as 0.1.

The studs or bolts are screwed up tightly along with a metal gasket or asbestos packing to provide a leak-proof joint between the cylinder and cylinder head. The tightness of the joint also depends upon the pitch of the bolts or studs, which should lie between 19\(\sqrt{d}\) to 28.5\(\sqrt{d}\). The pitch circle diameter (\(D_p\)) is usually taken as \(D + 3d\). The studs or bolts are designed in the same way as discussed above.

**Example 32.1.** A four stroke diesel engine has the following specifications:

Brake power = 5 kW; Speed = 1200 r.p.m.; Indicated mean effective pressure = 0.35 N/mm\(^2\);
Mechanical efficiency = 80%.

Determine: 1. bore and length of the cylinder; 2. thickness of the cylinder head; and 3. size of studs for the cylinder head.
Solution. Given: \(B.P. = 5\text{kW} = 5000 \text{ W} \); \(N = 1200 \text{ r.p.m. or } n = N/2 = 600 \); 
\(p_m = 0.35 \text{ N/mm}^2; \eta_m = 80\% = 0.8\)
1. **Bore and length of cylinder**

Let 
\(D = \text{Bore of the cylinder in mm,} \)
\(A = \text{Cross-sectional area of the cylinder} = \frac{\pi}{4} \times D^2 \text{ mm}^2 \)
\(l = \text{Length of the stroke in m.} \)
\(= 1.5 \text{ D mm} = 1.5 \text{ D / 1000 m} \)

(assume)

We know that the indicated power, 
\(I.P = B.P / \eta_m = 5000 / 0.8 = 6250 \text{ W} \)

We also know that the indicated power \((I.P.)\),
\[6250 = \frac{p_m \cdot l \cdot A \cdot n}{60} = \frac{0.35 \times 1.5D \times \pi D^2 \times 600}{60 \times 1000 \times 4} = 4.12 \times 10^{-3} D^3 \]

\((\therefore \text{ For four stroke engine, } n = N/2)\)

\[D^3 = 6250 / 4.12 \times 10^{-3} = 1517 \times 10^3 \text{ or } D = 115 \text{ mm} \text{ Ans.}\]

and 
\[l = 1.5 \times 115 = 172.5 \text{ mm} \]

Taking a clearance on both sides of the cylinder equal to 15\% of the stroke, therefore length of the cylinder,
\[L = 1.15 \times 172.5 = 198 \text{ say 200 mm} \text{ Ans.}\]

2. **Thickness of the cylinder head**

Since the maximum pressure \((p)\) in the engine cylinder is taken as 9 to 10 times the mean effective pressure \((p_m)\), therefore let us take
\[p = 9 \times p_m = 9 \times 0.35 = 3.15 \text{ N/mm}^2 \]

We know that thickness of the cylinder head,
\[t_h = D \sqrt[3]{\frac{C_p}{\sigma_t}} = 115 \sqrt[3]{\frac{0.1 \times 3.15}{42}} = 9.96 \text{ say 10 mm} \text{ Ans.}\]

(\(\text{Taking } C = 0.1 \text{ and } \sigma_t = 42 \text{ MPa} = 42 \text{ N/mm}^2\))

3. **Size of studs for the cylinder head**

Let 
\(d = \text{Nominal diameter of the stud in mm,} \)
\(d_c = \text{Core diameter of the stud in mm. It is usually taken as 0.84 } d. \)
\(\sigma_t = \text{Tensile stress for the material of the stud which is usually nickel steel.} \)
\(n_s = \text{Number of studs.} \)

We know that the force acting on the cylinder head (or on the studs)
\[= \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} (115)^2 \times 3.15 = 32702 \text{ N} \]

\((\therefore)\)

The number of studs \((n_s)\) are usually taken between 0.01 \(D + 4\) (i.e. 0.01 \(\times 115 + 4 = 5.15\)) and 0.02 \(D + 4\) (i.e. 0.02 \(\times 115 + 4 = 6.3\)). Let us take \(n_s = 6.\)

We know that resisting force offered by all the studs
\[= n_s \times \frac{\pi}{4} (d_c)^2 \sigma_t = 6 \times \frac{\pi}{4} (0.84d)^2 \times 65 = 216 d^2 \text{N} \]

(\(\therefore\) Taking \(\sigma_t = 65 \text{ MPa} = 65 \text{ N/mm}^2\))

From equations \((i)\) and \((ii)\),
\[d^2 = 32702 / 216 = 151 \text{ or } d = 12.3 \text{ say 14 mm} \]
The pitch circle diameter of the studs \((D_p)\) is taken as \(D + 3d\).

\[
D_p = 115 + 3 \times 14 = 157 \text{ mm}
\]

We know that the pitch of the studs

\[
\frac{\pi \times D_p}{n_s} = \frac{\pi \times 157}{6} = 82.2 \text{ mm}
\]

We know that for a leak-proof joint, the pitch of the studs should lie between \(19\sqrt{d}\) to \(28.5\sqrt{d}\), where \(d\) is the nominal diameter of the stud.

\[
\therefore \text{ Minimum pitch of the studs } = 19\sqrt{d} = 19\sqrt{14} = 71.1 \text{ mm}
\]

and maximum pitch of the studs

\[
= 28.5\sqrt{d} = 28.5\sqrt{14} = 106.6 \text{ mm}
\]

Since the pitch of the studs obtained above (i.e. 82.2 mm) lies within 71.1 mm and 106.6 mm, therefore, size of the stud \((d)\) calculated above is satisfactory.

\[
\therefore d = 14 \text{ mm} \quad \text{Ans.}
\]

### 32.5 Piston

The piston is a disc which reciprocates within a cylinder. It is either moved by the fluid or it moves the fluid which enters the cylinder. The main function of the piston of an internal combustion engine is to receive the impulse from the expanding gas and to transmit the energy to the crankshaft through the connecting rod. The piston must also disperse a large amount of heat from the combustion chamber to the cylinder walls.

![Fig. 32.3. Piston for I.C. engines (Trunk type).](image)
The piston of internal combustion engines are usually of trunk type as shown in Fig. 32.3. Such pistons are open at one end and consists of the following parts:

1. **Head or crown.** The piston head or crown may be flat, convex or concave depending upon the design of combustion chamber. It withstands the pressure of gas in the cylinder.

2. **Piston rings.** The piston rings are used to seal the cylinder in order to prevent leakage of the gas past the piston.

3. **Skirt.** The skirt acts as a bearing for the side thrust of the connecting rod on the walls of cylinder.

4. **Piston pin.** It is also called *gudgeon pin* or *wrist pin*. It is used to connect the piston to the connecting rod.

### 32.6 Design Considerations for a Piston

In designing a piston for I.C. engine, the following points should be taken into consideration:

1. It should have enormous strength to withstand the high gas pressure and inertia forces.
2. It should have minimum mass to minimise the inertia forces.
3. It should form an effective gas and oil sealing of the cylinder.
4. It should provide sufficient bearing area to prevent undue wear.
5. It should disperse the heat of combustion quickly to the cylinder walls.
6. It should have high speed reciprocation without noise.
7. It should be of sufficient rigid construction to withstand thermal and mechanical distortion.
8. It should have sufficient support for the piston pin.

### 32.7 Material for Pistons

The most commonly used materials for pistons of I.C. engines are cast iron, cast aluminium, forged aluminium, cast steel and forged steel. The cast iron pistons are used for moderately rated
engines with piston speeds below 6 m/s and aluminium alloy pistons are used for highly rated engines running at higher piston speeds. It may be noted that

1. Since the *coefficient of thermal expansion for aluminium is about 2.5 times that of cast iron, therefore, a greater clearance must be provided between the piston and the cylinder wall (than with cast iron piston) in order to prevent siezing of the piston when engine runs continuously under heavy loads. But if excessive clearance is allowed, then the piston will develop ‘piston slap’ while it is cold and this tendency increases with wear. The less clearance between the piston and the cylinder wall will lead to siezing of piston.

2. Since the aluminium alloys used for pistons have high **heat conductivity (nearly four times that of cast iron), therefore, these pistons ensure high rate of heat transfer and thus keeps down the maximum temperature difference between the centre and edges of the piston head or crown.

Notes: (a) For a cast iron piston, the temperature at the centre of the piston head ($T_C$) is about 425°C to 450°C under full load conditions and the temperature at the edges of the piston head ($T_E$) is about 200°C to 225°C.

(b) For aluminium alloy pistons, $T_C$ is about 260°C to 290°C and $T_E$ is about 185°C to 215°C.

3. Since the aluminium alloys are about ***three times lighter than cast iron, therefore, its mechanical strength is good at low temperatures, but they lose their strength (about 50%) at temperatures above 325°C. Sometimes, the pistons of aluminium alloys are coated with aluminium oxide by an electrical method.

32.8 Piston Head or Crown

The piston head or crown is designed keeping in view the following two main considerations, i.e.

1. It should have adequate strength to withstand the straining action due to pressure of explosion inside the engine cylinder, and

2. It should dissipate the heat of combustion to the cylinder walls as quickly as possible.

On the basis of first consideration of straining action, the thickness of the piston head is determined by treating it as a flat circular plate of uniform thickness, fixed at the outer edges and subjected to a uniformly distributed load due to the gas pressure over the entire cross-section.

The thickness of the piston head ($t_H$), according to Grashoff’s formula is given by

$$t_H = \frac{\sqrt{3pD^2}}{16\sigma_t} \text{ (in mm)} \quad \ldots(i)$$

where

- $p$ = Maximum gas pressure or explosion pressure in N/mm²,
- $D$ = Cylinder bore or outside diameter of the piston in mm, and
- $\sigma_t$ = Permissible bending (tensile) stress for the material of the piston in MPa or N/mm². It may be taken as 35 to 40 MPa for grey cast iron, 50 to 90 MPa for nickel cast iron and aluminium alloy and 60 to 100 MPa for forged steel.

On the basis of second consideration of heat transfer, the thickness of the piston head should be such that the heat absorbed by the piston due combustion of fuel is quickly transferred to the cylinder walls. Treating the piston head as a flat circular plate, its thickness is given by

$$t_H = \frac{H}{12.56k(T_C - T_E)} \text{ (in mm)} \quad \ldots(ii)$$

* The coefficient of thermal expansion for aluminium is $0.24 \times 10^{-6} \text{ m/°C}$ and for cast iron it is $0.1 \times 10^{-6} \text{ m/°C}$.

** The heat conductivity for aluminium is 174.75 W/m/°C and for cast iron it is 46.6 W/m/°C.

*** The density of aluminium is 2700 kg/m³ and for cast iron it is 7200 kg/m³.
where \( H \) = Heat flowing through the piston head in kJ/s or watts,  
\( k \) = Heat conductivity factor in W/m/°C. Its value is 46.6 W/m/°C for grey cast iron, 51.25 W/m/°C for steel and 174.75 W/m/°C for aluminium.

\( T_C \) = Temperature at the centre of the piston head in °C, and  
\( T_E \) = Temperature at the edges of the piston head in °C.

The temperature difference \((T_C - T_E)\) may be taken as 220°C for cast iron and 75°C for aluminium.

The heat flowing through the piston head \((H)\) may be determined by the following expression, \(i.e.,\),

\[
H = C \times HCV \times m \times B.P. \text{ (in kW)}
\]

where \( C \) = Constant representing that portion of the heat supplied to the engine which is absorbed by the piston. Its value is usually taken as 0.05.

\( HCV \) = Higher calorific value of the fuel in kJ/kg. It may be taken as \(45 \times 10^3\) kJ/kg for diesel and \(47 \times 10^3\) kJ/kg for petrol,

\( m \) = Mass of the fuel used in kg per brake power per second, and

\( B.P. \) = Brake power of the engine per cylinder

Notes: 1. The thickness of the piston head \((t_H)\) is calculated by using equations (i) and (ii) and larger of the two values obtained should be adopted.

2. When \( t_H \) is 6 mm or less, then no ribs are required to strengthen the piston head against gas loads. But when \( t_H \) is greater than 6 mm, then a suitable number of ribs at the centre line of the boss extending around the skirt should be provided to distribute the side thrust from the connecting rod and thus to prevent distortion of the skirt. The thickness of the ribs may be taken as \( t_H / 3 \) to \( t_H / 2 \).

3. For engines having length of stroke to cylinder bore \((L/D)\) ratio upto 1.5, a cup is provided in the top of the piston head with a radius equal to \(0.7D\). This is done to provide a space for combustion chamber.

32.9 Piston Rings

The piston rings are used to impart the necessary radial pressure to maintain the seal between the piston and the cylinder bore. These are usually made of grey cast iron or alloy cast iron because of their good wearing properties and also they retain spring characteristics even at high temperatures. The piston rings are of the following two types:

1. Compression rings or pressure rings, and
2. Oil control rings or oil scraper.

The compression rings or pressure rings are inserted in the grooves at the top portion of the piston and may be three to seven in number. These rings also transfer heat from the piston to the cylinder liner and absorb some part of the piston fluctuation due to the side thrust.

The oil control rings or oil scrapers are provided below the compression rings. These rings provide proper lubrication to the liner by allowing sufficient oil to move up during upward stroke and at the same time scraps the lubricating oil from the surface of the liner in order to minimise the flow of the oil to the combustion chamber.

The compression rings are usually made of rectangular cross-section and the diameter of the ring is slightly larger than the cylinder bore. A part of the ring is cut-off in order to permit it to go into the cylinder against the liner wall. The diagonal cut or step cut ends, as shown in Fig. 32.4 (a) and (b) respectively, may be used. The gap between the ends should be sufficiently large when the ring is put cold so that even at the highest temperature, the ends do not touch each other when the ring expands, otherwise there might be buckling of the ring.
The radial thickness \( t_1 \) of the ring may be obtained by considering the radial pressure between the cylinder wall and the ring. From bending stress consideration in the ring, the radial thickness is given by

\[
t_1 = D \sqrt{\frac{3p_w}{\sigma_t}}
\]

where

- \( D \) = Cylinder bore in mm,
- \( p_w \) = Pressure of gas on the cylinder wall in N/mm\(^2\). Its value is limited from 0.025 N/mm\(^2\) to 0.042 N/mm\(^2\), and
- \( \sigma_t \) = Allowable bending (tensile) stress in MPa. Its value may be taken from 85 MPa to 110 MPa for cast iron rings.

The axial thickness \( t_2 \) of the rings may be taken as 0.7 \( t_1 \) to \( t_1 \).

The minimum axial thickness \( t_2 \) may also be obtained from the following empirical relation:

\[
t_2 = \frac{D}{10n_R}
\]

where

- \( n_R \) = Number of rings.

The width of the top land (i.e. the distance from the top of the piston to the first ring groove) is made larger than other ring lands to protect the top ring from high temperature conditions existing at the top of the piston.

\[ b_1 = t_H \text{ to } 1.2t_H \]

The width of other ring lands (i.e. the distance between the ring grooves) in the piston may be made equal to or slightly less than the axial thickness of the ring \( t_2 \).

\[ b_2 = 0.75t_2 \text{ to } t_2 \]

The depth of the ring grooves should be more than the depth of the ring so that the ring does not take any piston side thrust.

The gap between the free ends of the ring is given by 3.5 \( t_1 \) to 4 \( t_1 \). The gap, when the ring is in the cylinder, should be 0.002 \( D \) to 0.004 \( D \).

**32.10 Piston Barrel**

It is a cylindrical portion of the piston. The maximum thickness \( t_3 \) of the piston barrel may be obtained from the following empirical relation:

\[
t_3 = 0.03D + b + 4.5 \text{ mm}
\]
where \( b \) = Radial depth of piston ring groove which is taken as 0.4 mm larger than the radial thickness of the piston ring \( (t_1) \)

\[ = t_1 + 0.4 \text{ mm} \]

Thus, the above relation may be written as

\[ t_3 = 0.03D + t_1 + 4.9 \text{ mm} \]

The piston wall thickness \( (t_4) \) towards the open end is decreased and should be taken as 0.25 \( t_3 \) to 0.35 \( t_3 \).

**32.11 Piston Skirt**

The portion of the piston below the ring section is known as *piston skirt*. It acts as a bearing for the side thrust of the connecting rod. The length of the piston skirt should be such that the bearing pressure on the piston barrel due to the side thrust does not exceed 0.25 N/mm\(^2\) of the projected area for low speed engines and 0.5 N/mm\(^2\) for high speed engines. It may be noted that the maximum thrust will be during the expansion stroke. The side thrust \( (R) \) on the cylinder liner is usually taken as 1/10 of the maximum gas load on the piston.

\[ \text{1000 cc twin -cylinder motorcycle engine.} \]

We know that maximum gas load on the piston,

\[ P = p \times \frac{\pi D^2}{4} \]

\[ \therefore \text{Maximum side thrust on the cylinder,} \]

\[ R = P/10 = 0.1 \times p \times \frac{\pi D^2}{4} \quad \ldots (i) \]

where \( p \) = Maximum gas pressure in N/mm\(^2\), and

\( D \) = Cylinder bore in mm.

The side thrust \( (R) \) is also given by

\[ R = \text{Bearing pressure} \times \text{Projected bearing area of the piston skirt} \]

\[ = p_b \times D \times l \]

where \( l \) = Length of the piston skirt in mm. \quad \ldots (ii)
From equations (i) and (ii), the length of the piston skirt \( l \) is determined. In actual practice, the length of the piston skirt is taken as 0.65 to 0.8 times the cylinder bore. Now the total length of the piston \( L \) is given by

\[
L = \text{Length of skirt} + \text{Length of ring section} + \text{Top land}
\]

The length of the piston usually varies between \( D \) and \( 1.5D \). It may be noted that a longer piston provides better bearing surface for quiet running of the engine, but it should not be made unnecessarily long as it will increase its own mass and thus the inertia forces.

### 32.12 Piston Pin

The piston pin (also called gudgeon pin or wrist pin) is used to connect the piston and the connecting rod. It is usually made hollow and tapered on the inside, the smallest inside diameter being at the centre of the pin, as shown in Fig. 32.5. The piston pin passes through the bosses provided on the inside of the piston skirt and the bush of the small end of the connecting rod. The centre of piston pin should be 0.02 \( D \) to 0.04 \( D \) above the centre of the skirt, in order to off-set the turning effect of the friction and to obtain uniform distribution of pressure between the piston and the cylinder liner.

The material used for the piston pin is usually case hardened steel alloy containing nickel, chromium, molybdenum or vanadium having tensile strength from 710 MPa to 910 MPa.

The connection between the piston pin and the small end of the connecting rod may be made either full floating type or semi-floating type. In the full floating type, the piston pin is free to turn both in the piston bosses and the bush of the small end of the connecting rod. The end movements of the piston pin should be secured by means of spring circlips, as shown in Fig. 32.6, in order to prevent the pin from touching and scoring the cylinder liner.

In the semi-floating type, the piston pin is either free to turn in the piston bosses and rigidly secured to the small end of the connecting rod, or it is free to turn in the bush of the small end of the connecting rod and is rigidly secured in the piston bosses by means of a screw, as shown in Fig. 32.7.

The piston pin should be designed for the maximum gas load or the inertia force of the piston, whichever is larger. The bearing area of the piston pin should be about equally divided between the piston pin bosses and the connecting rod bushing. Thus, the length of the pin in the connecting rod bushing will be about 0.45 of the cylinder bore or piston diameter \( D \), allowing for the end clearance.

* The mean diameter of the piston bosses is made 1.4 \( d_0 \) for cast iron pistons and 1.5 \( d_0 \) for aluminium pistons, where \( d_0 \) is the outside diameter of the piston pin. The piston bosses are usually tapered, increasing the diameter towards the piston wall.
of the pin etc. The outside diameter of the piston pin \((d_0)\) is determined by equating the load on the piston due to gas pressure \((p)\) and the load on the piston pin due to bearing pressure \((p_{b1})\) at the small end of the connecting rod bushing.

\[
\text{equation (i)}
\]

\[
\text{equation (ii)}
\]

From equations (i) and (ii), the outside diameter of the piston pin \((d_0)\) may be obtained.

The piston pin may be checked in bending by assuming the gas load to be uniformly distributed over the length \(l_1\) with supports at the centre of the bosses at the two ends. From Fig. 32.8, we find that the length between the supports,

\[
l_2 = l_1 + \frac{D - l_1}{2} = \frac{l_1 + D}{2}
\]

Now maximum bending moment at the centre of the pin,

\[
M = \frac{P}{2} \times \frac{l_2}{2} - \frac{P}{l_1} \times \frac{l_1}{2} \times \frac{l_1}{4}
\]

\[
= \frac{P}{2} \times \frac{l_2}{2} - \frac{P}{2} \times \frac{l_1}{4}
\]

\[
= \frac{P}{2} \left( \frac{l_1 + D}{2} \right) - \frac{P}{2} \times \frac{l_1}{4}
\]

\[
= \frac{P.l_1}{8} + \frac{P.D}{8} - \frac{P.l_1}{8} = \frac{P.D}{8}
\]
We have already discussed that the piston pin is made hollow. Let \( d_0 \) and \( d_i \) be the outside and inside diameters of the piston pin. We know that the section modulus,

\[
Z = \frac{\pi}{32} \left[ \frac{(d_0)^4 - (d_i)^4}{d_0} \right]
\]

We know that maximum bending moment,

\[
M = Z \times \sigma_b = \frac{\pi}{32} \left[ \frac{(d_0)^4 - (d_i)^4}{d_0} \right] \sigma_b
\]

where \( \sigma_b \) = Allowable bending stress for the material of the piston pin. It is usually taken as 84 MPa for case hardened carbon steel and 140 MPa for heat treated alloy steel.

Assuming \( d_i = 0.6 d_0 \), the induced bending stress in the piston pin may be checked.

### Example 32.2.

Design a cast iron piston for a single acting four stroke engine for the following data:

- Cylinder bore = 100 mm
- Stroke = 125 mm
- Maximum gas pressure = 5 N/mm²
- Indicated mean effective pressure = 0.75 N/mm²
- Mechanical efficiency = 80%
- Fuel consumption = 0.15 kg per brake power per hour
- Higher calorific value of fuel = 42 × 10³ kJ/kg
- Speed = 2000 r.p.m.

Any other data required for the design may be assumed.

**Solution.** Given : \( D = 100 \) mm ; \( L = 125 \) mm = 0.125 m ; \( p = 5 \) N/mm² ; \( p_m = 0.75 \) N/mm² ; \( \eta_{m} = 80\% = 0.8 \) ; \( m = 0.15 \) kg / BP / h = 41.7 × 10⁻⁶ kg / BP / s ; \( HCV = 42 \times 10^3 \) kJ / kg ; \( N = 2000 \) r.p.m.

The dimensions for various components of the piston are determined as follows :

1. **Piston head or crown**

   The thickness of the piston head or crown is determined on the basis of strength as well as on the basis of heat dissipation and the larger of the two values is adopted.
We know that the thickness of piston head on the basis of strength,
\[ t_H = \sqrt{\frac{3p_D}{16\sigma_t}} = \sqrt{\frac{3 \times 5(100)}{16 \times 38}} = 15.7 \text{ say } 16 \text{ mm} \]
...(Taking \( \sigma_t \) for cast iron = 38 MPa = 38 N/mm²)

Since the engine is a four stroke engine, therefore, the number of working strokes per minute,
\[ n = \frac{N}{2} = \frac{2000}{2} = 1000 \]
and cross-sectional area of the cylinder,
\[ A = \frac{\pi D^2}{4} = \frac{\pi (100)^2}{4} = 7855 \text{ mm}^2 \]

We know that indicated power,
\[ IP = \frac{p_m \cdot L \cdot A \cdot n}{60} = \frac{0.75 \times 0.125 \times 7855 \times 1000}{60} = 12270 \text{ W} \]
\[ = 12.27 \text{ kW} \]
\[ \therefore \text{Brake power,} \quad BP = IP \times \eta_m = 12.27 \times 0.8 = 9.8 \text{ kW} \]
...(\( \therefore \eta_m = BP / IP \))

We know that the heat flowing through the piston head,
\[ H = C \times HCV \times m \times BP \]
\[ = 0.05 \times 42 \times 10^3 \times 41.7 \times 10^{-6} \times 9.8 = 0.86 \text{ kW} = 860 \text{ W} \]
...(Taking \( C = 0.05 \))

\[ \therefore \text{Thickness of the piston head on the basis of heat dissipation,} \]
\[ t_H = \frac{C \times E}{12.56(k(T_C - T_E))} = \frac{860}{12.56 \times 46.6 \times 220} = 0.0067 \text{ m} = 6.7 \text{ mm} \]
...(\( \therefore \text{For cast iron, } k = 46.6 \text{ W/m/°C, and } T_C - T_E = 220^\circ \text{C} \))

Taking the larger of the two values, we shall adopt \( t_H = 16 \text{ mm} \) \text{Ans.}

Since the ratio of \( L / D \) is 1.25, therefore a cup in the top of the piston head with a radius equal to 0.7 \( D \) (i.e. 70 mm) is provided.

2. Radial ribs

The radial ribs may be four in number. The thickness of the ribs varies from \( t_H / 3 \) to \( t_H / 2 \).
\[ \therefore \text{Thickness of the ribs,} \quad t_R = \frac{16}{3} / \frac{16}{2} = 5.33 \text{ to } 8 \text{ mm} \]
Let us adopt \( t_R = 7 \text{ mm} \) \text{Ans.}

3. Piston rings

Let us assume that there are total four rings (i.e. \( n_r = 4 \)) out of which three are compression rings and one is an oil ring.

We know that the radial thickness of the piston rings,
\[ t_1 = D \sqrt{\frac{3p_w}{\sigma_t}} = 100 \sqrt{\frac{3 \times 0.035}{90}} = 3.4 \text{ mm} \]
...(Taking \( p_w = 0.035 \text{ N/mm}^2 \), and \( \sigma_t = 90 \text{ MPa} \))

and axial thickness of the piston rings
\[ t_2 = 0.7 \text{ to } t_1 = 0.7 \times 3.4 \text{ to } 3.4 \text{ mm} = 2.38 \text{ to } 3.4 \text{ mm} \]
Let us adopt \( t_2 = 3 \text{ mm} \)
We also know that the minimum axial thickness of the piston ring,

\[ t_2 = \frac{D}{10 n_r} = \frac{100}{10 \times 4} = 2.5 \text{ mm} \]

Thus the axial thickness of the piston ring as already calculated (i.e. \( t_2 = 3 \) mm) is satisfactory. Ans.

The distance from the top of the piston to the first ring groove, i.e. the width of the top land,

\[ b_1 = t_{H_1} = 16 \text{ to } 1.2 \times 16 \text{ mm} = 16 \text{ to } 19.2 \text{ mm} \]

and width of other ring lands,

\[ b_2 = 0.75 t_2 \text{ to } 3 \text{ mm} = 2.25 \text{ to } 3 \text{ mm} \]

Let us adopt \( b_1 = 18 \text{ mm} \); and \( b_2 = 2.5 \text{ mm} \) Ans.

We know that the gap between the free ends of the ring,

\[ G_1 = 3.5 t_1 \text{ to } 4 t_1 = 3.5 \times 3.4 \text{ to } 4 \times 3.4 \text{ mm} = 11.9 \text{ to } 13.6 \text{ mm} \]

and the gap when the ring is in the cylinder,

\[ G_2 = 0.002 D \text{ to } 0.004 D = 0.002 \times 100 \text{ to } 0.004 \times 100 \text{ mm} = 0.2 \text{ to } 0.4 \text{ mm} \]

Let us adopt \( G_1 = 12.8 \text{ mm} \); and \( G_2 = 0.3 \text{ mm} \) Ans.

\section*{4. Piston barrel}

Since the radial depth of the piston ring grooves (\( b \)) is about 0.4 mm more than the radial thickness of the piston rings (\( t_1 \)), therefore,

\[ b = t_1 + 0.4 = 3.4 + 0.4 = 3.8 \text{ mm} \]

We know that the maximum thickness of barrel,

\[ t_3 = 0.03 D + b + 4.5 \text{ mm} = 0.03 \times 100 + 3.8 + 4.5 = 11.3 \text{ mm} \]

and piston wall thickness towards the open end,

\[ t_4 = 0.25 t_3 \text{ to } 0.35 t_3 = 0.25 \times 11.3 \text{ to } 0.35 \times 11.3 = 2.8 \text{ to } 3.9 \text{ mm} \]

Let us adopt \( t_4 = 3.4 \text{ mm} \)

\section*{5. Piston skirt}

Let \( l = \) Length of the skirt in mm.

We know that the maximum side thrust on the cylinder due to gas pressure (\( p \)),

\[ R = \mu \times \frac{\pi D^2}{4} \times p = 0.1 \times \frac{\pi (100)^2}{4} \times 5 = 3928 \text{ N} \]

...(Taking \( \mu = 0.1 \))

We also know that the side thrust due to bearing pressure on the piston barrel (\( p_b \)),

\[ R = p_b \times D \times l = 0.45 \times 100 \times l = 45 l \text{ N} \]

...(Taking \( p_b = 0.45 \text{ N/mm}^2 \))

From above, we find that

\[ 45 l = 3928 \text{ or } l = 3928 / 45 = 87.3 \text{ say } 90 \text{ mm} \text{ Ans.} \]

\( \therefore \) Total length of the piston,

\[ L = \text{Length of the skirt} + \text{Length of the ring section} + \text{Top land} = l + (4 t_2 + 3 b_2) + b_1 = 90 + (4 \times 3 + 3 \times 3) + 18 = 129 \text{ say } 130 \text{ mm} \text{ Ans.} \]

\section*{6. Piston pin}

Let \( d_0 = \) Outside diameter of the pin in mm,

\[ l_1 = \text{Length of pin in the bush of the small end of the connecting rod in mm, and} \]
\[ p_{b1} = \text{Bearing pressure at the small end of the connecting rod bushing in N/mm}^2. \text{ It value for bronze bushing is taken as 25 N/mm}^2. \]

We know that load on the pin due to bearing pressure
\[ = \text{Bearing pressure} \times \text{Bearing area} = p_{b1} \times d_0 \times l_1 \]
\[ = 25 \times d_0 \times 0.45 \times 100 = 1125 \times d_0 \text{ N} \]
...(Taking \( l_1 = 0.45D \))

We also know that maximum load on the piston due to gas pressure or maximum gas load

\[ = \frac{\pi D^2}{4} \times p = \frac{\pi (100)^2}{4} \times 5 = 39275 \text{ N} \]

From above, we find that
\[ 1125 \times d_0 = 39275 \text{ or } d_0 = 39275 / 1125 = 34.9 \text{ say 35 mm Ans.} \]

The inside diameter of the pin \((d_i)\) is usually taken as 0.6 \(d_o\).
\[ \therefore d_i = 0.6 \times 35 = 21 \text{ mm Ans.} \]

Let the piston pin be made of heat treated alloy steel for which the bending stress \((\sigma_b)\) may be taken as 140 MPa. Now let us check the induced bending stress in the pin.

We know that maximum bending moment at the centre of the pin,
\[ M = \frac{PD}{8} = \frac{39275 \times 100}{8} = 491 \times 10^3 \text{ N-mm} \]

We also know that maximum bending moment \((M)\),
\[ 491 \times 10^3 = \frac{\pi}{32} \left[ \frac{(d_0)^4 - (d_i)^4}{d_0} \right] \sigma_b = \frac{\pi}{32} \left[ \frac{(35)^4 - (21)^4}{35} \right] \sigma_b = 3664 \sigma_b \]
\[ \therefore \sigma_b = 491 \times 10^3 / 3664 = 134 \text{ N/mm}^2 \text{ or MPa} \]

Since the induced bending stress in the pin is less than the permissible value of 140 MPa \(i.e. 140 \text{ N/mm}^2\), therefore, the dimensions for the pin as calculated above \(i.e. d_0 = 35 \text{ mm and } d_i = 21 \text{ mm}\) are satisfactory.

Air filter stops dust and dirt from being sucked into engine

Fan blows air over engine to cool it

Driveshaft

Disk brake

Spark plug

Twin rotors

German engineer Felix Wankel (1902-88) built a rotary engine in 1957. A triangular piston turns inside a chamber through the combustion cycle.
32.13 Connecting Rod

The connecting rod is the intermediate member between the piston and the crankshaft. Its primary function is to transmit the push and pull from the piston pin to the crankpin and thus convert the reciprocating motion of the piston into the rotary motion of the crank. The usual form of the connecting rod in internal combustion engines is shown in Fig. 32.9. It consists of a long shank, a small end and a big end. The cross-section of the shank may be rectangular, circular, tubular, \( I \)-section or \( H \)-section. Generally circular section is used for low speed engines while \( I \)-section is preferred for high speed engines.

![Fig. 32.9. Connecting rod.](image)

The *length of the connecting rod \( l \) depends upon the ratio of \( l / r \), where \( r \) is the radius of crank. It may be noted that the smaller length will decrease the ratio \( l / r \). This increases the angularity of the connecting rod which increases the side thrust of the piston against the cylinder liner which in turn increases the wear of the liner. The larger length of the connecting rod will increase the ratio \( l / r \). This decreases the angularity of the connecting rod and thus decreases the side thrust and the resulting wear of the cylinder. But the larger length of the connecting rod increases the overall height of the engine. Hence, a compromise is made and the ratio \( l / r \) is generally kept as 4 to 5.

The small end of the connecting rod is usually made in the form of an eye and is provided with a bush of phosphor bronze. It is connected to the piston by means of a piston pin.

The big end of the connecting rod is usually made split (in two **halves) so that it can be mounted easily on the crankpin bearing shells. The split cap is fastened to the big end with two cap bolts. The bearing shells of the big end are made of steel, brass or bronze with a thin lining (about 0.75 mm) of white metal or babbit metal. The wear of the big end bearing is allowed for by inserting thin metallic strips (known as *shims*) about 0.04 mm thick between the cap and the fixed half of the connecting rod. As the wear takes place, one or more strips are removed and the bearing is trued up.

* It is the distance between the centres of small end and big end of the connecting rod.

** One half is fixed with the connecting rod and the other half (known as cap) is fastened with two cap bolts.
The connecting rods are usually manufactured by drop forging process and it should have adequate strength, stiffness and minimum weight. The material mostly used for connecting rods varies from mild carbon steels (having 0.35 to 0.45 percent carbon) to alloy steels (chrome-nickel or chrome-molybdenum steels). The carbon steel having 0.35 percent carbon has an ultimate tensile strength of about 650 MPa when properly heat treated and a carbon steel with 0.45 percent carbon has a ultimate tensile strength of 750 MPa. These steels are used for connecting rods of industrial engines. The alloy steels have an ultimate tensile strength of about 1050 MPa and are used for connecting rods of aeroengines and automobile engines.

The bearings at the two ends of the connecting rod are either splash lubricated or pressure lubricated. The big end bearing is usually splash lubricated while the small end bearing is pressure lubricated. In the splash lubrication system, the cap at the big end is provided with a dipper or spout and set at an angle in such a way that when the connecting rod moves downward, the spout will dip into the lubricating oil contained in the sump. The oil is forced up the spout and then to the big end bearing. Now when the connecting rod moves upward, a splash of oil is produced by the spout. This splashed up lubricant finds its way into the small end bearing through the widely chamfered holes provided on the upper surface of the small end.

In the pressure lubricating system, the lubricating oil is fed under pressure to the big end bearing through the holes drilled in crankshaft, crankwebs and crank pin. From the big end bearing, the oil is fed to small end bearing through a fine hole drilled in the shank of the connecting rod. In some cases, the small end bearing is lubricated by the oil scrapped from the walls of the cylinder liner by the oil scraper rings.

### 32.14 Forces Acting on the Connecting Rod

The various forces acting on the connecting rod are as follows:

1. Force on the piston due to gas pressure and inertia of the reciprocating parts,
2. Force due to inertia of the connecting rod or inertia bending forces,
3. Force due to friction of the piston rings and of the piston, and
4. Force due to friction of the piston pin bearing and the crankpin bearing.

We shall now derive the expressions for the forces acting on a vertical engine, as discussed below.

1. **Force on the piston due to gas pressure and inertia of reciprocating parts**

   Consider a connecting rod $PC$ as shown in Fig. 32.10.
Let

\[ p = \text{Maximum pressure of gas,} \]
\[ D = \text{Diameter of piston,} \]
\[ A = \text{Cross-section area of piston} = \frac{\pi D^2}{4}, \]
\[ m_R = \text{Mass of reciprocating parts,} \]
\[ = \text{Mass of piston, gudgeon pin etc.} + \frac{1}{3} \text{rd mass of connecting rod}, \]
\[ \omega = \text{Angular speed of crank,} \]
\[ \phi = \text{Angle of inclination of the connecting rod with the line of stroke,} \]
\[ \theta = \text{Angle of inclination of the crank from top dead centre,} \]
\[ r = \text{Radius of crank,} \]
\[ l = \text{Length of connecting rod, and} \]
\[ n = \text{Ratio of length of connecting rod to radius of crank} = \frac{l}{r}. \]

We know that the force on the piston due to pressure of gas,
\[ F_L = \text{Pressure } \times \text{Area} = p \cdot A = p \times \frac{\pi D^2}{4} \]
and inertia force of reciprocating parts,
\[ F_I = \text{Mass } \times \text{Acceleration} = m_R \cdot \omega^2 \cdot r \left( \cos \theta + \frac{\cos 2\theta}{n} \right) \]

It may be noted that the inertia force of reciprocating parts opposes the force on the piston when it moves during its downward stroke (i.e., when the piston moves from the top dead centre to bottom dead centre). On the other hand, the inertia force of the reciprocating parts helps the force on the piston when it moves from the bottom dead centre to top dead centre.

\[ \therefore \text{Net force acting on the piston or piston pin (or gudgeon pin or wrist pin),} \]
\[ F_p = \text{Force due to gas pressure} \mp \text{Inertia force} \]
\[ = F_L \mp F_I \]

The –ve sign is used when piston moves from TDC to BDC and +ve sign is used when piston moves from BDC to TDC.

When weight of the reciprocating parts \((W_R = m_R \cdot g)\) is to be taken into consideration, then
\[ F_p = F_L \mp F_I \pm W_R \]

* Acceleration of reciprocating parts \(= \omega^2 \cdot r \left( \cos \theta + \frac{\cos 2\theta}{n} \right) \)
The force $F_p$ gives rise to a force $F_c$ in the connecting rod and a thrust $F_N$ on the sides of the cylinder walls. From Fig. 32.10, we see that force in the connecting rod at any instant,

$$F_c = \frac{F_p}{\cos \phi} = \frac{F_p}{\sqrt{1 - \sin^2 \frac{\theta}{n^2}}}$$

The force in the connecting rod will be maximum when the crank and the connecting rod are perpendicular to each other (i.e. when $\theta = 90^\circ$). But at this position, the gas pressure would be decreased considerably. Thus, for all practical purposes, the force in the connecting rod ($F_c$) is taken equal to the maximum force on the piston due to pressure of gas ($F_L$), neglecting piston inertia effects.

2. Force due to inertia of the connecting rod or inertia bending forces

Consider a connecting rod $PC$ and a crank $OC$ rotating with uniform angular velocity $\omega$ rad/s. In order to find the acceleration of various points on the connecting rod, draw the Klien’s acceleration diagram $CQNO$ as shown in Fig. 32.11 (a). $CO$ represents the acceleration of $C$ towards $O$ and $NO$ represents the acceleration of $P$ towards $O$. The acceleration of other points such as $D, E, F$ and $G$ etc., on the connecting rod $PC$ may be found by drawing horizontal lines from these points to intersect $CN$ at $d, e, f,$ and $g$ respectively. Now $dO, eO, fO$ and $gO$ represents the acceleration of $D, E, F$ and $G$ all towards $O$. The inertia force acting on each point will be as follows:

- Inertia force at $C = m \times \omega^2 \times CO$
- Inertia force at $D = m \times \omega^2 \times dO$
- Inertia force at $E = m \times \omega^2 \times eO$, and so on.

![Diagram of inertia forces](image-url)

The inertia forces will be opposite to the direction of acceleration or centrifugal forces. The inertia forces can be resolved into two components, one parallel to the connecting rod and the other perpendicular to rod. The parallel (or longitudinal) components adds up algebraically to the force

* For derivation, please refer to Authors’ popular book on ‘Theory of Machines’. 
acting on the connecting rod \( (F_C) \) and produces thrust on the pins. The perpendicular (or transverse) components produces bending action (also called whipping action) and the stress induced in the connecting rod is called **whipping stress**.

It may be noted that the perpendicular components will be maximum, when the crank and connecting rod are at right angles to each other.

The variation of the inertia force on the connecting rod is linear and is like a simply supported beam of variable loading as shown in Fig. 32.11 (b) and (c). Assuming that the connecting rod is of uniform cross-section and has mass \( m_1 \) kg per unit length, therefore,

Inertia force per unit length at the crankpin
\[
= m_1 \times \omega^2 r
\]
and inertia force per unit length at the piston pin
\[
= 0
\]
Inertia force due to small element of length \( dx \) at a distance \( x \) from the piston pin \( P \),
\[
dF_1 = m_1 \times \omega^2 r \times \frac{x}{l} \times dx
\]
\[
\therefore \text{Resultant inertia force},
F_1 = \int_0^l m_1 \times \omega^2 r \times \frac{x}{l} \times dx = \frac{m_1 \times \omega^2 r}{l} \left[ \frac{x^2}{2} \right]_0^l
\]
\[
= \frac{m_1 l}{2} \times \omega^2 r = \frac{m}{2} \times \omega^2 r \quad \text{(Substituting } m_1 \cdot l = m)\]
This resultant inertia force acts at a distance of \( 2l/3 \) from the piston pin \( P \).

Since it has been assumed that \( \frac{1}{3} \) rd mass of the connecting rod is concentrated at piston pin \( P \) (i.e. small end of connecting rod) and \( \frac{2}{3} \) rd at the crankpin (i.e. big end of connecting rod), therefore, the reaction at these two ends will be in the same proportion. i.e.
\[
R_P = \frac{1}{3} F_1, \text{ and } R_C = \frac{2}{3} F_1
\]
Now the bending moment acting on the rod at section $X - X$ at a distance $x$ from $P$:

$$M_X = R_P \times x - m_1 \times \frac{\omega^2 r \times x}{l} \times \frac{1}{2} x \times \frac{x}{3}$$

$$= \frac{1}{3} F_1 \times x - \frac{m_1 l^2}{2} \times \frac{\omega^2 r \times x^3}{3l^2}$$

...(Multiplying and dividing the latter expression by $l$)

$$= \frac{F_1 \times x}{3} - \frac{F_1 \times x^3}{3l^2} = \frac{F_1}{3} \left( x - \frac{x^3}{l^2} \right)$$

...(i)

For maximum bending moment, differentiate $M_X$ with respect to $x$ and equate to zero, i.e.

$$\frac{dM_X}{dx} = 0 \text{ or } \frac{F_1}{3} \left[ 1 - \frac{3x^2}{l^2} \right] = 0$$

$$\therefore \quad 1 - \frac{3x^2}{l^2} = 0 \text{ or } 3x^2 = l^2 \text{ or } x = \frac{l}{\sqrt{3}}$$

Maximum bending moment,

$$M_{\text{max}} = \frac{F_1}{3} \left[ \frac{l}{\sqrt{3}} - \left( \frac{l}{3\sqrt{3}} \right)^3 \right]$$

...[From equation (i)]

$$= \frac{F_1}{3} \left[ \frac{l}{\sqrt{3}} - \frac{l}{3\sqrt{3}} \right] = \frac{F_1 \times l}{3\sqrt{3}} \times \frac{2}{3} = \frac{2F_1 \times l}{9\sqrt{3}}$$

$$= 2 \times \frac{m}{2} \times \omega^2 r \times \frac{l}{9\sqrt{3}} = m \times \omega^2 r \times \frac{l}{9\sqrt{3}}$$

and the maximum bending stress, due to inertia of the connecting rod,

$$\sigma_{\text{max}} = \frac{M_{\text{max}}}{Z}$$

where

$$Z = \text{Section modulus.}$$

From above we see that the maximum bending moment varies as the square of speed, therefore, the bending stress due to high speed will be dangerous. It may be noted that the maximum axial force and the maximum bending stress do not occur simultaneously. In an I.C. engine, the maximum gas load occurs close to top dead centre whereas the maximum bending stress occurs when the crank angle $\theta = 65^\circ$ to $70^\circ$ from top dead centre. The pressure of gas falls suddenly as the piston moves from dead centre. Thus the general practice is to design a connecting rod by assuming the force in the connecting rod ($F_C$) equal to the maximum force due to pressure ($F_L$), neglecting piston inertia effects and then checked for bending stress due to inertia force (i.e. whipping stress).

**3. Force due to friction of piston rings and of the piston**

The frictional force ($F$) of the piston rings may be determined by using the following expression:

$$F = \pi D \cdot t_R \cdot n_R \cdot P_R \cdot \mu$$

where

- $D =$ Cylinder bore,
- $t_R =$ Axial width of rings,
- $n_R =$ Number of rings,
- $P_R =$ Pressure of gas,
- $\mu =$ Coefficient of friction.

* B.M. due to variable force from $0$ to $m_0 \omega^2 r \times l$ is equal to the area of triangle multiplied by the distance of C.G. from $X - X \left( \text{i.e. } \frac{x}{3} \right)$. 
A Textbook of Machine Design

\[ n_R = \text{Number of rings}, \]
\[ P_R = \text{Pressure of rings (0.025 to 0.04 N/mm}^2\text{)}, \text{and} \]
\[ \mu = \text{Coefficient of friction (about 0.1)}. \]

Since the frictional force of the piston rings is usually very small, therefore, it may be neglected.

The friction of the piston is produced by the normal component of the piston pressure which varies from 3 to 10 percent of the piston pressure. If the coefficient of friction is about 0.05 to 0.06, then the frictional force due to piston will be about 0.5 to 0.6 of the piston pressure, which is very low. Thus, the frictional force due to piston is also neglected.

4. **Force due to friction of the piston pin bearing and crankpin bearing**

The force due to friction of the piston pin bearing and crankpin bearing, is to bend the connecting rod and to increase the compressive stress on the connecting rod due to the direct load. Thus, the maximum compressive stress in the connecting rod will be

\[ \sigma_{c (max)} = \text{Direct compressive stress + Maximum bending or whipping stress due to inertia bending stress} \]

32.15 Design of Connecting Rod

In designing a connecting rod, the following dimensions are required to be determined:

1. Dimensions of cross-section of the connecting rod,
2. Dimensions of the crankpin at the big end and the piston pin at the small end,
3. Size of bolts for securing the big end cap, and
4. Thickness of the big end cap.

The procedure adopted in determining the above mentioned dimensions is discussed as below:

---

This experimental car burns hydrogen fuel in an ordinary piston engine. Its exhaust gases cause no pollution, because they contain only water vapour.
1. Dimensions of cross-section of the connecting rod

A connecting rod is a machine member which is subjected to alternating direct compressive and tensile forces. Since the compressive forces are much higher than the tensile forces, therefore, the cross-section of the connecting rod is designed as a strut and the Rankine’s formula is used.

A connecting rod, as shown in Fig. 32.12, subjected to an axial load $W$ may buckle with $X$-axis as neutral axis (i.e. in the plane of motion of the connecting rod) or $Y$-axis as neutral axis (i.e. in the plane perpendicular to the plane of motion). The connecting rod is considered like both ends hinged for buckling about $X$-axis and both ends fixed for buckling about $Y$-axis.

A connecting rod should be equally strong in buckling about both the axes.

Let

$A$ = Cross-sectional area of the connecting rod, \\
$l$ = Length of the connecting rod, \\
$\sigma_c$ = Compressive yield stress, \\
$W_B$ = Buckling load, \\
$I_{xx}$ and $I_{yy}$ = Moment of inertia of the section about $X$-axis and $Y$-axis respectively, and \\
$k_{xx}$ and $k_{yy}$ = Radius of gyration of the section about $X$-axis and $Y$-axis respectively.

According to Rankine’s formula,

$W_B$ about $X$–axis = \[
\frac{\sigma_c A}{1 + a \left( \frac{l}{k_{xx}} \right)^2} = \frac{\sigma_c A}{1 + a \left( \frac{l}{k_{xx}} \right)^2} \quad \ldots (\because \text{For both ends hinged, } L = l)
\]

and $W_B$ about $Y$–axis = \[
\frac{\sigma_c A}{1 + a \left( \frac{l}{k_{yy}} \right)^2} = \frac{\sigma_c A}{1 + a \left( \frac{l}{2k_{yy}} \right)^2} \quad \ldots [\because \text{For both ends fixed, } L = \frac{l}{2}]
\]

where

$L$ = Equivalent length of the connecting rod, and \\
$a$ = Constant \\
$= 1/7500$, for mild steel \\
$= 1/9000$, for wrought iron \\
$= 1/1600$, for cast iron

\[\text{Fig. 32.12. Buckling of connecting rod.}\]

In order to have a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal, i.e.

\[
\frac{\sigma_c A}{1 + a \left( \frac{l}{k_{xx}} \right)^2} = \frac{\sigma_c A}{1 + a \left( \frac{l}{k_{yy}} \right)^2} \quad \text{or} \quad \left( \frac{l}{k_{xx}} \right)^2 = \left( \frac{l}{2k_{yy}} \right)^2
\]

\[\because \quad k_{xx}^2 = 4k_{yy}^2 \quad \text{or} \quad l_{xx} = 4l_{yy} \quad \ldots (\because I = A \cdot k^2)\]
This shows that the connecting rod is four times strong in buckling about Y-axis than about X-axis. If $I_{xx} > 4I_{yy}$, then buckling will occur about Y-axis and if $I_{xx} < 4I_{yy}$, buckling will occur about X-axis. In actual practice, $I_{xx}$ is kept slightly less than $4I_{yy}$. It is usually taken between 3 and 3.5 and the connecting rod is designed for bucking about X-axis. The design will always be satisfactory for buckling about Y-axis.

The most suitable section for the connecting rod is I-section with the proportions as shown in Fig. 32.13 (a).

Let thickness of the flange and web of the section = $t$

Width of the section, $B = 4t$

and depth or height of the section, $H = 5t$

From Fig. 32.13 (a), we find that area of the section, $A = 2(4t \times t) + 3t \times t = 11t^2$

Moment of inertia of the section about X-axis,

$$I_{xx} = \frac{1}{12} \left[ 4t \left(5t\right)^3 - 3t \left(3t\right)^3 \right] = \frac{419}{12} t^4$$

and moment of inertia of the section about Y-axis,

$$I_{yy} = \left[ 2 \times \frac{1}{12} t \times (4t)^3 + \frac{1}{12} (3t)^3 \right] = \frac{131}{12} t^4$$

$$\therefore \quad \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since the value of $\frac{I_{xx}}{I_{yy}}$ lies between 3 and 3.5, therefore, I-section chosen is quite satisfactory.

After deciding the proportions for I-section of the connecting rod, its dimensions are determined by considering the buckling of the rod about X-axis (assuming both ends hinged) and applying the Rankine’s formula. We know that buckling load,

$$W_B = \frac{\sigma_c A}{1 + \left(\frac{L}{k_{xx}}\right)^2}$$

The buckling load ($W_B$) may be calculated by using the following relation, i.e.

$$W_B = \text{Max. gas force} \times \text{Factor of safety}$$

The factor of safety may be taken as 5 to 6.

Notes:

(a) The I-section of the connecting rod is used due to its lightness and to keep the inertia forces as low as possible specially in case of high speed engines. It can also withstand high gas pressure.

(b) Sometimes a connecting rod may have rectangular section. For slow speed engines, circular section may be used.

(c) Since connecting rod is manufactured by forging, therefore the sharp corner of I-section are rounded off as shown in Fig. 32.13 (b) for easy removal of the section from dies.
The dimensions $B = 4t$ and $H = 5t$, as obtained above by applying the Rankine’s formula, are at the middle of the connecting rod. The width of the section ($B$) is kept constant throughout the length of the connecting rod, but the depth or height varies. The depth near the small end (or piston end) is taken as $H_1 = 0.75H$ to $0.9H$ and the depth near the big end (or crank end) is taken $H_2 = 1.1H$ to $1.25H$.

2. Dimensions of the crankpin at the big end and the piston pin at the small end

Since the dimensions of the crankpin at the big end and the piston pin (also known as gudgeon pin or wrist pin) at the small end are limited, therefore, fairly high bearing pressures have to be allowed at the bearings of these two pins.

The crankpin at the big end has removable precision bearing shells of brass or bronze or steel with a thin lining (1 mm or less) of bearing metal (such as tin, lead, babbit, copper, lead) on the inner surface of the shell. The allowable bearing pressure on the crankpin depends upon many factors such as material of the bearing, viscosity of the lubricating oil, method of lubrication and the space limitations. The value of bearing pressure may be taken as 7 N/mm$^2$ to 12.5 N/mm$^2$ depending upon the material and method of lubrication used.

The piston pin bearing is usually a phosphor bronze bush of about 3 mm thickness and the allowable bearing pressure may be taken as 10.5 N/mm$^2$ to 15 N/mm$^2$.

Since the maximum load to be carried by the crankpin and piston pin bearings is the maximum force in the connecting rod ($F_C$), therefore the dimensions for these two pins are determined for the maximum force in the connecting rod ($F_C$) which is taken equal to the maximum force on the piston due to gas pressure ($F_L$) neglecting the inertia forces.

We know that maximum gas force,

$$ F_L = \frac{\pi D^2}{4} \times p \quad \ldots (i) $$

where

- $D$ = Cylinder bore or piston diameter in mm, and
- $p$ = Maximum gas pressure in N/mm$^2$

Now the dimensions of the crankpin and piston pin are determined as discussed below:

Let

- $d_c$ = Diameter of the crank pin in mm,
- $l_c$ = Length of the crank pin in mm,
- $p_{bc}$ = Allowable bearing pressure in N/mm$^2$, and
- $d_p, l_p, p_{bp}$ = Corresponding values for the piston pin.

We know that load on the crank pin

$$ = \text{Projected area} \times \text{Bearing pressure} $$

$$ = d_c \cdot l_c \cdot p_{bc} \quad \ldots (ii) $$

Similarly, load on the piston pin

$$ = d_p \cdot l_p \cdot p_{bp} \quad \ldots (iii) $$
Equating equations (i) and (ii), we have

\[ F_L = d_c \cdot l_c \cdot p_{bc} \]

Taking \( l_c = 1.25 \, d_c \) to 1.5 \( d_c \), the value of \( d_c \) and \( l_c \) are determined from the above expression.

Again, equating equations (i) and (iii), we have

\[ F_L = d_p \cdot l_p \cdot p_{bp} \]

Taking \( l_p = 1.5 \, d_p \) to 2 \( d_p \), the value of \( d_p \) and \( l_p \) are determined from the above expression.

**3. Size of bolts for securing the big end cap**

The bolts and the big end cap are subjected to tensile force which corresponds to the inertia force of the reciprocating parts at the top dead centre on the exhaust stroke. We know that inertia force of the reciprocating parts is

\[ F_I = m_R \cdot \omega^2 \cdot r \left( \cos \theta + \frac{\cos 2\theta}{lr} \right) \]

We also know that at the top dead centre, the angle of inclination of the crank with the line of stroke, \( \theta = 0 \)

\[ F_I = m_R \cdot \omega^2 \cdot r \left( 1 + \frac{r}{l} \right) \]

where
- \( m_R \) = Mass of the reciprocating parts in kg,
- \( \omega \) = Angular speed of the engine in rad / s,
- \( r \) = Radius of the crank in metres, and
- \( l \) = Length of the connecting rod in metres.

The bolts may be made of high carbon steel or nickel alloy steel. Since the bolts are under repeated stresses but not alternating stresses, therefore, a factor of safety may be taken as 6.

Let
- \( d_{cb} \) = Core diameter of the bolt in mm,
- \( \sigma_t \) = Allowable tensile stress for the material of the bolts in MPa, and
- \( n_b \) = Number of bolts. Generally two bolts are used.

\[ \therefore \text{Force on the bolts} = \frac{\pi}{4} (d_{cb})^2 \sigma_t \times n_b \]
Equating the inertia force to the force on the bolts, we have

\[ F_I = \frac{\pi}{4} (d_{cb})^2 \sigma_t \times n_b \]

From this expression, \( d_{cb} \) is obtained. The nominal or major diameter \( (d_b) \) of the bolt is given by

\[ d_b = \frac{d_{cb}}{0.84} \]

4. **Thickness of the big end cap**

   The thickness of the big end cap \( (t_c) \) may be determined by treating the cap as a beam freely supported at the cap bolt centres and loaded by the inertia force at the top dead centre on the exhaust stroke \( (i.e. F_I \text{ when } \theta = 0) \). This load is assumed to act in between the uniformly distributed load and the centrally concentrated load. Therefore, the maximum bending moment acting on the cap will be taken as

\[ M_C = \frac{F_I \times x}{6} \]

where

- \( x \) = Distance between the bolt centres.
- \( = \text{Dia. of crankpin or big end bearing } (d_c) + 2 \times \text{Thickness of bearing liner (3 mm)} + \text{Clearance (3 mm)} \)

Let

- \( b_c = \text{Width of the cap in mm. It is equal to the length of the crankpin or big end bearing } (l_c) \), and
- \( \sigma_b = \text{Allowable bending stress for the material of the cap in MPa.} \)

We know that section modulus for the cap,

\[ Z_C = \frac{b_c (t_c)^2}{6} \]

∴ Bending stress,

\[ \sigma_b = \frac{M_C}{Z_C} = \frac{F_I \times x}{6} \times \frac{6}{b_c (t_c)^2} = \frac{F_I \times x}{b_c (t_c)^2} \]

From this expression, the value of \( t_c \) is obtained.

**Note:** The design of connecting rod should be checked for whipping stress \( (i.e. \text{bending stress due to inertia force on the connecting rod}).\)

**Example 32.3.** Design a connecting rod for an I.C. engine running at 1800 r.p.m. and developing a maximum pressure of 3.15 N/mm². The diameter of the piston is 100 mm; mass of the reciprocating parts per cylinder 2.25 kg; length of connecting rod 380 mm; stroke of piston 190 mm and compression ratio 6 : 1. Take a factor of safety of 6 for the design. Take length to diameter ratio for big end bearing as 1.3 and small end bearing as 2 and the corresponding bearing pressures as 10 N/mm² and 15 N/mm². The density of material of the rod may be taken as 8000 kg/m³ and the allowable stress in the bolts as 60 N/mm² and in cap as 80 N/mm². The rod is to be of I-section for which you can choose your own proportions.

Draw a neat dimensioned sketch showing provision for lubrication. Use Rankine formula for which the numerator constant may be taken as 320 N/mm² and the denominator constant \( 1 / 7500 \).

* We know that the maximum bending moment for a simply or freely supported beam with a uniformly distributed load of \( F_I \) over a length \( x \) between the supports (In this case, \( x \) is the distance between the cap bolt centres) is \( \frac{F_I \times x}{8} \). When the load \( F_I \) is assumed to act at the centre of the freely supported beam, then the maximum bending moment is \( \frac{F_I \times x}{4} \). Thus the maximum bending moment in between these two bending moments \( (i.e. \frac{F_I \times x}{8} \text{ and } \frac{F_I \times x}{4}) \) is \( \frac{F_I \times x}{6} \).
Solution. Given : \( N = 1800 \text{ r.p.m.} \); \( p = 3.15 \text{ N/mm}^2 \); \( D = 100 \text{ mm} \); \( m_R = 2.25 \text{ kg} \); \( l = 380 \text{ mm} \); \( \text{Stroke} = 190 \text{ mm} \); Compression ratio = 6 : 1; \( F. S. = 6 \).

The connecting rod is designed as discussed below:

1. **Dimension of I- section of the connecting rod**

Let us consider an I-section of the connecting rod, as shown in Fig. 32.14(a), with the following proportions:

- Flange and web thickness of the section = \( t \)
- Width of the section, \( B = 4t \)
- Depth or height of the section, \( H = 5t \)

First of all, let us find whether the section chosen is satisfactory or not.

We have already discussed that the connecting rod is considered like both ends hinged for buckling about \( X \)-axis and both ends fixed for buckling about \( Y \)-axis. The connecting rod should be equally strong in buckling about both the axes. We know that in order to have a connecting rod equally strong about both the axes,

\[
I_{xx} = 4I_{yy}
\]

where
\[
I_{xx} = \text{Moment of inertia of the section about } X\text{-axis, and}
I_{yy} = \text{Moment of inertia of the section about } Y\text{-axis.}
\]

In actual practice, \( I_{xx} \) is kept slightly less than 4 \( I_{yy} \). It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about \( X \)-axis.

Now, for the section as shown in Fig. 32.14(a), area of the section,

\[
A = 2(4t \times t) + 3t \times t = 11t^2
\]

\[
I_{xx} = \frac{1}{12} \left[ 4t(5t)^3 - 3t \times (3t)^3 \right] = \frac{419}{12} t^4
\]

and

\[
I_{yy} = 2 \times \frac{1}{12} \times t(4t)^3 + \frac{1}{12} \times 3t \times t^3 = \frac{131}{12} t^4
\]

\[
\therefore \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2
\]

Since \( \frac{I_{xx}}{I_{yy}} = 3.2 \), therefore the section chosen is quite satisfactory.

Now let us find the dimensions of this I-section. Since the connecting rod is designed by taking the force on the connecting rod \( (F_C) \) equal to the maximum force on the piston \( (F_L) \) due to gas pressure, therefore,

\[
F_C = F_L = \frac{\pi D^2}{4} \times p = \frac{\pi(100)^2}{4} \times 3.15 = 24,740 \text{ N}
\]

We know that the connecting rod is designed for buckling about \( X \)-axis (i.e. in the plane of motion of the connecting rod) assuming both ends hinged. Since a factor of safety is given as 6, therefore the buckling load,

\[
W_B = F_C \times F. S. = 24,740 \times 6 = 148,440 \text{ N}
\]

* Superfluous data
We know that radius of gyration of the section about X-axis,

\[ k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{419 t^4}{12} \times \frac{1}{11 t^2}} = 1.78 t \]

Length of crank,

\[ r = \frac{\text{Stroke of piston}}{2} = \frac{190}{2} = 95 \text{ mm} \]

Length of the connecting rod,

\[ l = 380 \text{ mm} \]

\[ \therefore \text{Equivalent length of the connecting rod for both ends hinged,} \]

\[ L = l = 380 \text{ mm} \]

Now according to Rankine’s formula, we know that buckling load \( (W_B) \),

\[
\frac{148440}{320} = \frac{\sigma_c A}{1 + a \left( \frac{L}{k_{xx}} \right)^2} = \frac{320 \times 11 t^2}{1 + \frac{1}{7500} \left( \frac{380}{1.78 t} \right)^2}
\]

\[ ... \text{(It is given that } \sigma_c = 320 \text{ MPa or N/mm}^2 \text{ and } a = 1 / 7500) \]

\[
\frac{148440}{1 + \frac{6.1}{t^2}} = \frac{11 t^2}{t^2 + 6.1}
\]

\[ 464 (t^2 + 6.1) = 11 t^4 \]

or

\[ t^4 - 42.2 t^2 - 257.3 = 0 \]

\[ \therefore \quad t^2 = \frac{42.2 \pm \sqrt{(42.2)^2 + 4 \times 257.3}}{2} = \frac{42.2 \pm 53}{2} = 47.6 \]

or

\[ t = 6.9 \text{ say } 7 \text{ mm} \]

Thus, the dimensions of I-section of the connecting rod are:

Thickness of flange and web of the section

\[ = t = 7 \text{ mm} \text{ Ans.} \]

Width of the section, \( B = 4 t = 4 \times 7 = 28 \text{ mm} \text{ Ans.} \)

and depth or height of the section,

\[ H = 5 t = 5 \times 7 = 35 \text{ mm} \text{ Ans.} \]
These dimensions are at the middle of the connecting rod. The width \(B\) is kept constant throughout the length of the rod, but the depth \(H\) varies. The depth near the big end or crank end is kept as \(1.1H\) to \(1.25H\) and the depth near the small end or piston end is kept as \(0.75H\) to \(0.9H\). Let us take

Depth near the big end,
\[ H_1 = 1.2H = 1.2 \times 35 = 42 \text{ mm} \]
and depth near the small end,
\[ H_2 = 0.85H = 0.85 \times 35 = 29.75 \text{ say 30 mm} \]

\[ \therefore \text{Dimensions of the section near the big end} = 42 \text{ mm} \times 28 \text{ mm Ans.} \]
and dimensions of the section near the small end
\[ = 30 \text{ mm} \times 28 \text{ mm Ans.} \]

Since the connecting rod is manufactured by forging, therefore the sharp corners of \(I\)-section are rounded off, as shown in Fig. 32.14 \((b)\), for easy removal of the section from the dies.

2. Dimensions of the crankpin or the big end bearing and piston pin or small end bearing

Let
\[ d_c = \text{Diameter of the crankpin or big end bearing,} \]
\[ l_c = \text{Length of the crankpin or big end bearing} = 1.3 d_c \quad \text{...(Given)} \]
\[ p_{bc} = \text{Bearing pressure} = 10 \text{ N/mm}^2 \quad \text{...(Given)} \]

We know that load on the crankpin or big end bearing
\[ = \text{Projected area} \times \text{Bearing pressure} \]
\[ = d_c \times l_c \times p_{bc} = d_c \times 1.3 d_c \times 10 = 13 (d_c)^2 \]

Since the crankpin or the big end bearing is designed for the maximum gas force \(F_L\), therefore, equating the load on the crankpin or big end bearing to the maximum gas force, \(i.e.\)
\[ 13 (d_c)^2 = F_L = 24 \, 740 \text{ N} \]
\[ \therefore (d_c)^2 = 24 \, 740 / 13 = 1903 \text{ or } d_c = 43.6 \text{ say 44 mm Ans.} \]
and
\[ l_c = 1.3 d_c = 1.3 \times 44 = 57.2 \text{ say 58 mm Ans.} \]

The big end has removable precision bearing shells of brass or bronze or steel with a thin lining (1mm or less) of bearing metal such as babbit.

Again, let
\[ d_p = \text{Diameter of the piston pin or small end bearing,} \]
\[ l_p = \text{Length of the piston pin or small end bearing} = 2d_p \quad \text{...(Given)} \]
\[ p_{bp} = \text{Bearing pressure} = 15 \text{ N/mm}^2 \quad \text{...(Given)} \]

We know that the load on the piston pin or small end bearing
\[ = \text{Projected area} \times \text{Bearing pressure} \]
\[ = d_p \times l_p \times p_{bp} = d_p \times 2 d_p \times 15 = 30 (d_p)^2 \]

Since the piston pin or the small end bearing is designed for the maximum gas force \(F_L\), therefore, equating the load on the piston pin or the small end bearing to the maximum gas force, \(i.e.\)
\[ 30 (d_p)^2 = 24 \, 740 \text{ N} \]
\[ \therefore (d_p)^2 = 24 \, 740 / 30 = 825 \text{ or } d_p = 28.7 \text{ say 29 mm Ans.} \]
and
\[ l_p = 2 d_p = 2 \times 29 = 58 \text{ mm Ans.} \]

The small end bearing is usually a phosphor bronze bush of about 3 mm thickness.
3. Size of bolts for securing the big end cap

Let \( d_{ch} \) = Core diameter of the bolts,
\( \sigma_t \) = Allowable tensile stress for the material of the bolts
\( = 60 \text{ N/mm}^2 \) ...(Given)

and \( n_b \) = Number of bolts. Generally two bolts are used.

We know that force on the bolts
\[
F_1 = \frac{\pi}{4} (d_{cb})^2 \sigma_t \times n_b = \frac{\pi}{4} (d_{cb})^2 \times 60 \times 2 = 94.26 (d_{cb})^2
\]

The bolts and the big end cap are subjected to tensile force which corresponds to the inertia force of the reciprocating parts at the top dead centre on the exhaust stroke. We know that inertia force,\( F_I = m_R \cdot R \cdot \omega^2 \cdot \theta \cos \theta / 2 \cos \theta / l_r \theta \)+
\[
= 9490 \text{ N}
\]

Equating the inertia force to the force on the bolts, we have
\[
9490 = 94.26 (d_{cb})^2 \text{ or } (d_{cb})^2 = 9490 / 94.26 = 100.7
\]

\[
\therefore d_{cb} = 10.03 \text{ mm}
\]

and nominal diameter of the bolt,
\[
d_b = \frac{d_{cb}}{0.84} = \frac{10.03}{0.84} = 11.94 \text{ say 12 mm Ans.}
\]

4. Thickness of the big end cap

Let \( t_c \) = Thickness of the big end cap,
\( b_c \) = Width of the big end cap. It is taken equal to the length of the crankpin or big end bearing (\( l_c \))
\( = 58 \text{ mm} \) (calculated above)
\( \sigma_b \) = Allowable bending stress for the material of the cap
\( = 80 \text{ N/mm}^2 \) ...(Given)

The big end cap is designed as a beam freely supported at the cap bolt centres and loaded by the inertia force at the top dead centre on the exhaust stroke \( (i.e. F_1 \text{ when } \theta = 0) \). Since the load is assumed to act in between the uniformly distributed load and the centrally concentrated load, therefore, maximum bending moment is taken as
\[
M_C = \frac{F_1 \times x}{6}
\]

where \( x \) = Distance between the bolt centres
A Textbook of Machine Design

$$= \text{Dia. of crank pin or big end bearing} + 2 \times \text{Thickness of bearing liner} + \text{Nominal dia. of bolt} + \text{Clearance}$$

$$= (d_c + 2 \times 3 + d_b + 3) \text{ mm} = 44 + 6 + 12 + 3 = 65 \text{ mm}$$

∴ Maximum bending moment acting on the cap,

$$M_C = \frac{F_i \times x}{6} = \frac{9490 \times 65}{6} = 102810 \text{ N-mm}$$

Section modulus for the cap

$$Z_C = \frac{b_c (t_c)^2}{6} = \frac{58(t_c)^2}{6} = 9.7 (t_c)^2$$

We know that bending stress ($\sigma_b$),

$$\frac{80 = M_C}{Z_C} = \frac{102810}{9.7 (t_c)^2} = \frac{10600}{(t_c)^2}$$

∴ $$(t_c)^2 = \frac{10600}{80} = 132.5 \text{ or } t_c = 11.5 \text{ mm Ans.}$$

Let us now check the design for the induced bending stress due to inertia bending forces on the connecting rod (i.e. whipping stress).

We know that mass of the connecting rod per metre length,

$$m_1 = \text{Volume} \times \text{density} = \text{Area} \times \text{length} \times \text{density}$$

$$= A \times l \times \rho = 11r^2 \times l \times \rho$$

$$= 11(0.007)^2 (0.38) 8000 = 1.64 \text{ kg}$$

$$\text{[} \therefore \rho = 8000 \text{ kg/m}^3 \text{ (given)}\text{]}$$

∴ Maximum bending moment,

$$M_{max} = m \cdot \omega^2 \cdot r \times \frac{l}{9\sqrt{3}} = m_1 \cdot \omega^2 \cdot r \times \frac{l^2}{9\sqrt{3}}$$

$$= 1.64 \left(\frac{2\pi \times 1800}{60}\right)^2 (0.095) \left(\frac{0.38}{9\sqrt{3}}\right)^2 = 51.3 \text{ N-m}$$

$$= 51300 \text{ N-mm}$$

and section modulus,

$$Z_{xx} = \frac{I_{xx}}{5t/2} = \frac{419 t^4}{12} \times \frac{2}{5t} = \frac{13.97 t^3}{3} = 13.97 \times 7^3 = 4792 \text{ mm}^3$$

∴ Maximum bending stress (induced) due to inertia bending forces or whipping stress,

$$\sigma_{b(max)} = \frac{M_{max}}{Z_{xx}} = \frac{51300}{4792} = 10.7 \text{ N/mm}^2$$

Since the maximum bending stress induced is less than the allowable bending stress of 80 N/mm², therefore the design is safe.
32.16 Crankshaft

A crankshaft \( i.e. \) a shaft with a crank) is used to convert reciprocating motion of the piston into rotatory motion or vice versa. The crankshaft consists of the shaft parts which revolve in the main bearings, the crankpins to which the big ends of the connecting rod are connected, the crank arms or webs (also called cheeks) which connect the crankpins and the shaft parts. The crankshaft, depending upon the position of crank, may be divided into the following two types:

1. Side crankshaft or overhung crankshaft, as shown in Fig. 32.15 (a), and
2. Centre crankshaft, as shown in Fig. 32.15 (b).

32.17 Material and manufacture of Crankshafts

The crankshafts are subjected to shock and fatigue loads. Thus material of the crankshaft should be tough and fatigue resistant. The crankshafts are generally made of carbon steel, special steel or special cast iron.

In industrial engines, the crankshafts are commonly made from carbon steel such as 40 C 8, 55 C 8 and 60 C 4. In transport engines, manganese steel such as 20 Mn 2, 27 Mn 2 and 37 Mn 2 are generally used for the making of crankshaft. In aero engines, nickel chromium steel such as 35 Ni 1 Cr 60 and 40 Ni 2 Cr 1 Mo 28 are extensively used for the crankshaft.

The crankshafts are made by drop forging or casting process but the former method is more common. The surface of the crankpin is hardened by case carburizing, nitriding or induction hardening.

32.18 Bearing Pressures and Stresses in Crankshaft

The bearing pressures are very important in the design of crankshafts. The \(^*\) maximum permissible bearing pressure depends upon the maximum gas pressure, journal velocity, amount and method of lubrication and change of direction of bearing pressure.

The following two types of stresses are induced in the crankshaft.

1. Bending stress; and 2. Shear stress due to torsional moment on the shaft.

\(^*\) The values of maximum permissible bearing pressures for different types of engines are given in Chapter 26, Table 26.3.
Most crankshaft failures are caused by a progressive fracture due to repeated bending or reversed torsional stresses. Thus the crankshaft is under fatigue loading and, therefore, its design should be based upon endurance limit. Since the failure of a crankshaft is likely to cause a serious engine destruction and neither all the forces nor all the stresses acting on the crankshaft can be determined accurately, therefore a high factor of safety from 3 to 4, based on the endurance limit, is used.

The following table shows the allowable bending and shear stresses for some commonly used materials for crankshafts:

<table>
<thead>
<tr>
<th>Material</th>
<th>Bending limit in MPa</th>
<th>Allowable stress in MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chrome nickel</td>
<td>525</td>
<td>130 to 175</td>
</tr>
<tr>
<td>Carbon steel and cast steel</td>
<td>225</td>
<td>56 to 75</td>
</tr>
<tr>
<td>Alloy cast iron</td>
<td>140</td>
<td>35 to 47</td>
</tr>
</tbody>
</table>

### 32.19 Design Procedure for Crankshaft

1. First of all, find the magnitude of the various loads on the crankshaft.
2. Determine the distances between the supports and their position with respect to the loads.
3. For the sake of simplicity and also for safety, the shaft is considered to be supported at the centres of the bearings and all the forces and reactions to be acting at these points. The distances between the supports depend on the length of the bearings, which in turn depend on the diameter of the shaft because of the allowable bearing pressures.
4. The thickness of the cheeks or webs is assumed to be from 0.4 \( d_s \) to 0.6 \( d_s \), where \( d_s \) is the diameter of the shaft. It may also be taken as 0.22\( D \) to 0.32\( D \), where \( D \) is the bore of cylinder in mm.
5. Now calculate the distances between the supports.
6. Assuming the allowable bending and shear stresses, determine the main dimensions of the crankshaft.

**Notes:**
1. The crankshaft must be designed or checked for at least two crank positions. Firstly, when the crankshaft is subjected to maximum bending moment and secondly when the crankshaft is subjected to maximum twisting moment or torque.
2. The additional moment due to weight of flywheel, belt tension and other forces must be considered.
3. It is assumed that the effect of bending moment does not exceed two bearings between which a force is considered.

### 32.20 Design of Centre Crankshaft

We shall design the centre crankshaft by considering the two crank positions, *i.e.* when the crank is at dead centre (or when the crankshaft is subjected to maximum bending moment) and when the crank is at angle at which the twisting moment is maximum. These two cases are discussed in detail as below:

1. **When the crank is at dead centre.** At this position of the crank, the maximum gas pressure on the piston will transmit maximum force on the crankpin in the plane of the crank causing only bending of the shaft. The crankpin as well as ends of the crankshaft will be only subjected to bending moment. Thus, when the crank is at the dead centre, the bending moment on the shaft is maximum and the twisting moment is zero.
Consider a single throw three bearing crankshaft as shown in Fig. 32.16.

Let

\[ D = \text{Piston diameter or cylinder bore in mm}, \]
\[ p = \text{Maximum intensity of pressure on the piston in N/mm}^2, \]
\[ W = \text{Weight of the flywheel acting downwards in N, and} \]
\[ T_1 + T_2 = \text{Resultant belt tension or pull acting horizontally in N}. \]

The thrust in the connecting rod will be equal to the gas load on the piston \( F_p \). We know that gas load on the piston,

\[ F_p = \frac{\pi}{4} \times D^2 \times p \]

Due to this piston gas load \( F_p \), acting horizontally, there will be two horizontal reactions \( H_1 \) and \( H_2 \) at bearings 1 and 2 respectively, such that

\[ H_1 = \frac{F_p \times b_1}{b}; \quad \text{and} \quad H_2 = \frac{F_p \times b_2}{b} \]

Due to the weight of the flywheel \( W \) acting downwards, there will be two vertical reactions \( V_2 \) and \( V_3 \) at bearings 2 and 3 respectively, such that

\[ V_2 = \frac{W \times c_1}{c} ; \quad \text{and} \quad V_3 = \frac{W \times c_2}{c} \]

Now due to the resultant belt tension \((T_1 + T_2)\), acting horizontally, there will be two horizontal reactions \( H'_2 \) and \( H'_3 \) at bearings 2 and 3 respectively, such that

\[ H'_2 = \frac{(T_1 + T_2) \times c_1}{c} ; \quad \text{and} \quad H'_3 = \frac{(T_1 + T_2) \times c_2}{c} \]

The resultant force at bearing 2 is given by

\[ R_2 = \sqrt{(H'_2)^2 + (V_2)^2} \]

* \( T_1 \) is the belt tension in the tight side and \( T_2 \) is the belt tension in the slack side.
and the resultant force at bearing 3 is given by

\[ R_3 = \sqrt{(H_3)^2 + (V_3)^2} \]

Now the various parts of the centre crankshaft are designed for bending only, as discussed below:

(a) Design of crankpin

Let
- \( d_c \) = Diameter of the crankpin in mm,
- \( l_c \) = Length of the crankpin in mm,
- \( \sigma_b \) = Allowable bending stress for the crankpin in N/mm².

We know that bending moment at the centre of the crankpin,

\[ M_C = H_1 \cdot b_2 \]  

... (i)

We also know that

\[ M_C = \frac{\pi}{32} (d_c)^3 \sigma_b \]  

... (ii)

From equations (i) and (ii), diameter of the crankpin is determined. The length of the crankpin is given by

\[ l_c = \frac{F_p}{d_c \cdot \rho_b} \]

where
- \( \rho_b \) = Permissible bearing pressure in N/mm².

(b) Design of left hand crank web

The crank web is designed for eccentric loading. There will be two stresses acting on the crank web, one is direct compressive stress and the other is bending stress due to piston gas load (\( F_p \)).
The thickness \((t)\) of the crank web is given empirically as
\[
t = 0.4 \cdot ds \text{ to } 0.6 \cdot ds
\]
\[
= 0.22D \text{ to } 0.32D
\]
\[
= 0.65 \cdot dc + 6.35 \text{ mm}
\]
where
\[
ds = \text{Shaft diameter in mm},
\]
\[
D = \text{Bore diameter in mm}, \text{ and}
\]
\[
dc = \text{Crankpin diameter in mm},
\]
The width of crank web \((w)\) is taken as
\[
w = 1.125 \cdot dc + 12.7 \text{ mm}
\]
We know that maximum bending moment on the crank web,
\[
M = H_1 \left( b_2 - \frac{l}{2} - \frac{t}{2} \right)
\]
and section modulus,
\[
Z = \frac{1}{6} \times w \cdot t^2
\]
∴ Bending stress,
\[
\sigma_b = \frac{M}{Z} = \frac{6H_1 \left( b_2 - \frac{l}{2} - \frac{t}{2} \right)}{w \cdot t^2}
\]
and direct compressive stress on the crank web,
\[
\sigma_c = \frac{H_1}{w \cdot t}
\]
∴ Total stress on the crank web
\[
= \text{Bending stress} + \text{Direct stress} = \sigma_b + \sigma_c
\]
\[
= \frac{6H_1 \left( b_2 - \frac{l}{2} - \frac{t}{2} \right)}{w \cdot t^2} + \frac{H_1}{w \cdot t}
\]
This total stress should be less than the permissible bending stress.

(c) **Design of right hand crank web**

The dimensions of the right hand crank web \((i.e.\) thickness and width\) are made equal to left hand crank web from the balancing point of view.

(d) **Design of shaft under the flywheel**

Let
\[
ds = \text{Diameter of shaft in mm}.
\]
We know that bending moment due to the weight of flywheel,
\[
M_w = V_3 \cdot c_1
\]
and bending moment due to belt tension,
\[
M_T = H_3' \cdot c_1
\]
These two bending moments act at right angles to each other. Therefore, the resultant bending moment at the flywheel location,
\[
M_s = (M_w)^2 + (M_T)^2 = (V_3 \cdot c_1)^2 + (H_3' \cdot c_1)^2
\]
We also know that the bending moment at the shaft,
\[
M_s = \frac{\pi}{32} (d_s)^3 \sigma_b
\]
where
\[
\sigma_b = \text{Allowable bending stress in N/mm}^2.
\]
From equations \((i)\) and \((ii)\), we may determine the shaft diameter \((d_s)\).
2. When the crank is at an angle of maximum twisting moment

The twisting moment on the crankshaft will be maximum when the tangential force on the crank ($F_T$) is maximum. The maximum value of tangential force lies when the crank is at angle of 25° to 30° from the dead centre for a constant volume combustion engines (i.e., petrol engines) and 30° to 40° for constant pressure combustion engines (i.e., diesel engines).

Consider a position of the crank at an angle of maximum twisting moment as shown in Fig. 32.17 (a). If $p'$ is the intensity of pressure on the piston at this instant, then the piston gas load at this position of crank,

$$F_p = \frac{\pi}{4} D^2 p'$$

and thrust on the connecting rod,

$$F_Q = \frac{F_p}{\cos \phi}$$

where $\phi = \text{Angle of inclination of the connecting rod with the line of stroke} \ PO$.

The $F_Q$ thrust in the connecting rod may be divided into two components, one perpendicular to the crank and the other along the crank. The component of $F_Q$ perpendicular to the crank is the tangential force ($F_T$) and the component of $F_Q$ along the crank is the radial force ($F_R$) which produces thrust on the crankshaft bearings. From Fig. 32.17 (b), we find that

$$F_T = F_Q \sin (\theta + \phi)$$

and

$$F_R = F_Q \cos (\theta + \phi)$$

It may be noted that the tangential force will cause twisting of the crankpin and shaft while the radial force will cause bending of the shaft.

* For further details, see Author’s popular book on ‘Theory of Machines’.
Due to the tangential force \(F_T\), there will be two reactions at bearings 1 and 2, such that

\[ H_{T1} = \frac{F_T \times b_1}{b}; \quad \text{and} \quad H_{T2} = \frac{F_T \times b_2}{b} \]

Due to the radial force \(F_R\), there will be two reactions at the bearings 1 and 2, such that

\[ H_{R1} = \frac{F_R \times b_1}{b}; \quad \text{and} \quad H_{R2} = \frac{F_R \times b_2}{b} \]

The reactions at the bearings 2 and 3, due to the flywheel weight \(W\) and resultant belt pull \((T_1 + T_2)\) will be same as discussed earlier.

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let \(d_c\) = Diameter of the crankpin in mm.

We know that bending moment at the centre of the crankpin,

\[ M_C = H_{R1} \times b_2 \]

and twisting moment on the crankpin,

\[ T_C = H_{T1} \times r \]

\[ \therefore \] Equivalent twisting moment on the crankpin,

\[ T_e = \sqrt{(M_C)^2 + (T_C)^2} = \sqrt{(H_{R1} \times b_2)^2 + (H_{T1} \times r)^2} \quad \ldots (i) \]

We also know that twisting moment on the crankpin,

\[ T_e = \frac{\pi}{16} (d_c)^3 \tau \quad \ldots (ii) \]

where \(\tau\) = Allowable shear stress in the crankpin.

From equations \((i)\) and \((ii)\), the diameter of the crankpin is determined.
(b) Design of shaft under the flywheel

Let \( d_s \) = Diameter of the shaft in mm.

We know that bending moment on the shaft,
\[ M_S = R_3 \times c_1 \]
and twisting moment on the shaft,
\[ T_S = F_T \times r \]
\[ \therefore \quad \text{Equivalent twisting moment on the shaft,} \]
\[ T_e = \sqrt{(M_S)^2 + (T_S)^2} = \sqrt{(R_3 \times c_1)^2 + (F_T \times r)^2} \] \( \ldots \) (i)

We also know that equivalent twisting moment on the shaft,
\[ T_e = \frac{\pi}{16} (d_s)^3 \tau \] \( \ldots \) (ii)

where \( \tau \) = Allowable shear stress in the shaft.

From equations (i) and (ii), the diameter of the shaft is determined.

(c) Design of shaft at the juncture of right hand crank arm

Let \( d_{s1} \) = Diameter of the shaft at the juncture of right hand crank arm.

We know that bending moment at the juncture of the right hand crank arm,
\[ M_{S1} = R_1 \left[ b_2 + \frac{l_c}{2} + \frac{t}{2} \right] - F_Q \left( \frac{l_c}{2} + \frac{t}{2} \right) \]
and the twisting moment at the juncture of the right hand crank arm,
\[ T_{S1} = F_T \times r \]
\[ \therefore \quad \text{Equivalent twisting moment at the juncture of the right hand crank arm,} \]
\[ T_e = \sqrt{(M_{S1})^2 + (T_{S1})^2} \] \( \ldots \) (i)

We also know that equivalent twisting moment,
\[ T_e = \frac{\pi}{16} (d_{s1})^3 \tau \] \( \ldots \) (ii)

where \( \tau \) = Allowable shear stress in the shaft.

From equations (i) and (ii), the diameter of the shaft at the juncture of the right hand crank arm is determined.

(d) Design of right hand crank web

The right hand crank web is subjected to the following stresses:

(i) Bending stresses in two planes normal to each other, due to the radial and tangential components of \( F_Q \),
(ii) Direct compressive stress due to \( F_R \), and
(iii) Torsional stress.

The bending moment due to the radial component of \( F_Q \) is given by,
\[ M_R = H_{R2} \left( b_h - \frac{l_c}{2} - \frac{t}{2} \right) \] \( \ldots \) (i)

We also know that \[ M_R = \sigma_{bh} \times Z = \sigma_{bh} \times \frac{1}{6} \times w \cdot t^2 \] \( \ldots \) (ii)

where \( \sigma_{br} \) = Bending stress in the radial direction, and

\[ Z = \text{Section modulus} = \frac{1}{6} \times w \cdot r^2 \]

From equations (i) and (ii), the value of bending stress \( \sigma_{br} \) is determined.

The bending moment due to the tangential component of \( F_Q \) is maximum at the juncture of crank and shaft. It is given by

\[ M_T = F_T \left[ r - \frac{d_{j1}}{2} \right] \quad \text{... (iii)} \]

where

\( d_{j1} \) = Shaft diameter at juncture of right hand crank arm, i.e. at bearing 2.

We also know that

\[ M_T = \sigma_{bt} \times Z = \sigma_{bt} \times \frac{1}{6} \times t \cdot w^2 \quad \text{... (iv)} \]

where \( \sigma_{bt} \) = Bending stress in tangential direction.

From equations (iii) and (iv), the value of bending stress \( \sigma_{bt} \) is determined.

The direct compressive stress is given by,

\[ \sigma_d = \frac{F_R}{2w.t} \]

The maximum compressive stress (\( \sigma_c \)) will occur at the upper left corner of the cross-section of the crank.

\[ \therefore \sigma_c = \sigma_{br} + \sigma_{bt} + \sigma_d \]

Now, the twisting moment on the arm,

\[ T = H_{T1} \left( b_2 + \frac{l_c}{2} \right) - F_T \times \frac{l_c}{2} = H_{T2} \left( b_1 - \frac{l_c}{2} \right) \]

We know that shear stress on the arm,

\[ \tau = \frac{T}{Z_p} = \frac{4.5 T}{w \cdot t^2} \]

where \( Z_p \) = Polar section modulus = \( \frac{w \cdot t^2}{4.5} \)

\[ \therefore \text{Maximum or total combined stress}, \quad (\sigma_c)_{max} = \frac{\sigma_c}{2} + \frac{1}{2} \sqrt{(\sigma_c)^2 + 4\tau^2} \]
The value of \( (\sigma_c)_{\text{max}} \) should be within safe limits. If it exceeds the safe value, then the dimension \( w \) may be increased because it does not affect other dimensions.

(e) **Design of left hand crank web**

Since the left hand crank web is not stressed to the extent as the right hand crank web, therefore, the dimensions for the left hand crank web may be made same as for right hand crank web.

(f) **Design of crankshaft bearings**

The bearing 2 is the most heavily loaded and should be checked for the safe bearing pressure.

We know that the total reaction at the bearing 2,

\[
R_2 = \frac{F_p}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2}
\]

\[
\therefore \text{Total bearing pressure} = \frac{R_2}{l_2 \cdot d_{sl}}
\]

where \( l_2 \) = Length of bearing 2.

### 32.21 Side or Overhung Crankshaft

The side or overhung crankshafts are used for medium size and large horizontal engines. Its main advantage is that it requires only two bearings in either the single or two crank construction. The design procedure for the side or overhung crankshaft is same as that for centre crankshaft. Let us now design the side crankshaft by considering the two crank positions, *i.e.* when the crank is at dead centre (or when the crankshaft is subjected to maximum bending moment) and when the crank is at an angle at which the twisting moment is maximum. These two cases are discussed in detail as below:

1. **When the crank is at dead centre.** Consider a side crankshaft at dead centre with its loads and distances of their application, as shown in Fig. 32.18.

![Fig. 32.18. Side crankshaft at dead centre.](image)
Let 

\[ D = \text{Piston diameter or cylinder bore in mm}, \]
\[ p = \text{Maximum intensity of pressure on the piston in N/mm}^2, \]
\[ W = \text{Weight of the flywheel acting downwards in N}, \]
\[ T_1 + T_2 = \text{Resultant belt tension or pull acting horizontally in N}. \]

We know that gas load on the piston,

\[ F_P = \frac{\pi}{4} D^2 p \]

Due to this piston gas load \(F_P\) acting horizontally, there will be two horizontal reactions \(H_1\) and \(H_2\) at bearings 1 and 2 respectively, such that

\[ H_1 = \frac{F_P (a + b)}{b}; \quad \text{and} \quad H_2 = \frac{F_P a}{b} \]

Due to the weight of the flywheel \(W\) acting downwards, there will be two vertical reactions \(V_1\) and \(V_2\) at bearings 1 and 2 respectively, such that

\[ V_1 = \frac{W b_1}{b}; \quad \text{and} \quad V_2 = \frac{W b_2}{b} \]

Now due to the resultant belt tension \((T_1 + T_2)\) acting horizontally, there will be two horizontal reactions \(H_1'\) and \(H_2'\) at bearings 1 and 2 respectively, such that

\[ H_1' = \frac{(T_1 + T_2)b_1}{b}; \quad \text{and} \quad H_2' = \frac{(T_1 + T_2)b_2}{b} \]

The various parts of the side crankshaft, when the crank is at dead centre, are now designed as discussed below:

**(a) Design of crankpin.** The dimensions of the crankpin are obtained by considering the crankpin in bearing and then checked for bending stress.

Let

\[ d_c = \text{Diameter of the crankpin in mm}, \]
\[ l_c = \text{Length of the crankpin in mm}, \]
\[ p_b = \text{Safe bearing pressure on the pin in N/mm}^2. \]

We know that \(F_P = d_c \cdot l_c \cdot p_b\).

From this expression, the values of \(d_c\) and \(l_c\) may be obtained. The length of crankpin is usually from 0.6 to 1.5 times the diameter of pin.

The crankpin is now checked for bending stress. If it is assumed that the crankpin acts as a cantilever and the load on the crankpin is uniformly distributed, then maximum bending moment will be \(\frac{F_p \times l_c}{2}\). But in actual practice, the bearing pressure on the crankpin is not uniformly distributed and may, therefore, give a greater value of bending moment ranging between \(\frac{F_p \times l_c}{2}\) and \(F_p \times l_c\). So, a mean value of bending moment, \(i.e.\frac{3}{4} F_p \times l_c\) may be assumed.
A Textbook of Machine Design

**Maximum bending moment at the crankpin,**

\[ M = \frac{3}{4} F_p l_c \]  
\[ \text{... (Neglecting pin collar thickness)} \]

**Section modulus for the crankpin,**

\[ Z = \frac{\pi}{32} (d_c)^3 \]

**Bending stress induced,**

\[ \sigma_b = \frac{M}{Z} \]

This induced bending stress should be within the permissible limits.

**(b) Design of bearings.** The bending moment at the centre of the bearing 1 is given by

\[ M = F_p (0.75 l_c + t + 0.5 l_1) \]  
\[ \text{...(i)} \]

where

- \( l_c \) = Length of the crankpin,
- \( t \) = Thickness of the crank web = 0.45 \( d_c \) to 0.75 \( d_c \), and
- \( l_1 \) = Length of the bearing = 1.5 \( d_c \) to 2 \( d_c \).

We also know that

\[ M = \frac{\pi}{32} (d_1)^3 \sigma_b \]  
\[ \text{...(ii)} \]

From equations (i) and (ii), the diameter of the bearing 1 may be determined.

**Note:** The bearing 2 is also made of the same diameter. The length of the bearings are found on the basis of allowable bearing pressures and the maximum reactions at the bearings.

**(c) Design of crank web.** When the crank is at dead centre, the crank web is subjected to a bending moment and to a direct compressive stress.

We know that bending moment on the crank web,

\[ M = F_p (0.75 l_c + 0.5 t) \]

and section modulus, \( Z = \frac{1}{6} w \cdot t^2 \)

**Bending stress,**

\[ \sigma_b = \frac{M}{Z} \]

We also know that direct compressive stress,

\[ \sigma_d = \frac{F_p}{w \cdot t} \]

**Total stress on the crank web,**

\[ \sigma_T = \sigma_b + \sigma_d \]

This total stress should be less than the permissible limits.

**(d) Design of shaft under the flywheel.** The total bending moment at the flywheel location will be the resultant of horizontal bending moment due to the gas load and belt pull and the vertical bending moment due to the flywheel weight.

Let \( d_s \) = Diameter of shaft under the flywheel.

We know that horizontal bending moment at the flywheel location due to piston gas load,

\[ M_1 = F_p (a + b_2) - H_1 \cdot b_2 = H_2 \cdot b_2 \]
and horizontal bending moment at the flywheel location due to belt pull,

\[ M_2 = H_1 \cdot b_2 = H_2 \cdot b_1 = \frac{(T_1 + T_2) b_1 b_2}{b} \]

\[ \therefore \text{Total horizontal bending moment,} \]

\[ M_{H} = M_1 + M_2 \]

We know that vertical bending moment due to flywheel weight,

\[ M_v = V_1 b_2 = V_2 b_1 = \frac{W b_1 b_2}{b} \]

\[ \therefore \text{Resultant bending moment,} \]

\[ M_R = \sqrt{(M_{H})^2 + (M_v)^2} \]

We also know that

\[ M_R = \frac{\pi}{32} (d_s)^3 \sigma_b \]

From equations (i) and (ii), the diameter of shaft \((d_s)\) may determined.

2. **When the crank is at an angle of maximum twisting moment.** Consider a position of the crank at an angle of maximum twisting moment as shown in Fig. 32.19. We have already discussed in the design of a centre crankshaft that the thrust in the connecting rod \((F_Q)\) gives rise to the tangential force \((F_T)\) and the radial force \((F_R)\).

Due to the tangential force \((F_T)\), there will be two reactions at the bearings 1 and 2, such that

\[ H_{T1} = \frac{F_T (a + b)}{b}; \quad \text{and} \quad H_{T2} = \frac{F_T \times a}{b} \]

Due to the radial force \((F_R)\), there will be two reactions at the bearings 1 and 2, such that

\[ H_{R1} = \frac{F_R (a + b)}{b}; \quad \text{and} \quad H_{R2} = \frac{F_R \times a}{b} \]

The reactions at the bearings 1 and 2 due to the flywheel weight \((W)\) and resultant belt pull \((T_1 + T_2)\) will be same as discussed earlier.

Now the various parts of the crankshaft are designed as discussed below:

(a) **Design of crank web.** The most critical section is where the web joins the shaft. This section is subjected to the following stresses:

(i) Bending stress due to the tangential force \(F_T\);
(iii) Bending stress due to the radial force \( F_R \);
(iv) Direct compressive stress due to the radial force \( F_R \); and

We know that bending moment due to the tangential force,

\[
M_{BT} = F_T \left( r - \frac{d_1}{2} \right)
\]

where \( d_1 = \text{Diameter of the bearing 1} \).

\[
\therefore \text{Bending stress due to the tangential force,} \quad \sigma_{BT} = \frac{M_{BT}}{Z} = \frac{6M_{BT}}{t \cdot w^2}
\]

...(\( \because Z = \frac{1}{6} \times t \cdot w^2 \)) ...(i)

We know that bending moment due to the radial force,

\[
M_{BR} = F_R \left( 0.75 l_c + 0.5 t \right)
\]

\[
\therefore \text{Bending stress due to the radial force,} \quad \sigma_{BR} = \frac{M_{BR}}{Z} = \frac{6M_{BR}}{w \cdot t^2}
\]

...(Here \( Z = \frac{1}{6} \times w \cdot t^2 \)) ...(ii)

We know that direct compressive stress,

\[
\sigma_d = \frac{F_R}{w \cdot t}
\]

...(iii)

\[
\therefore \text{Total compressive stress,} \quad \sigma_c = \sigma_{BT} + \sigma_{BR} + \sigma_d
\]

...(iv)

We know that twisting moment due to the tangential force,

\[
T = F_T \left( 0.75 l_c + 0.5 t \right)
\]

\[
\therefore \text{Shear stress,} \quad \tau = \frac{T}{Z_P} = \frac{4.5T}{w \cdot t^2}
\]

where \( Z_P = \text{Polar section modulus} = \frac{w \cdot t^2}{4.5} \)
Now the total or maximum stress is given by
\[ \sigma_{\text{max}} = \frac{\sigma_c}{2} + \frac{1}{2} \sqrt{(\sigma_c)^2 + 4\tau^2} \]  
...(v)

This total maximum stress should be less than the maximum allowable stress.

(b) Design of shaft at the junction of crank

Let \( d_{s1} \) = Diameter of the shaft at the junction of the crank.

We know that bending moment at the junction of the crank,
\[ M = F_Q (0.75l_c + t) \]

and twisting moment on the shaft
\[ T = F_T \times r \]

\[ \therefore \text{Equivalent twisting moment,} \]
\[ T_e = \sqrt{M^2 + T^2} \]  
...(i)

We also know that equivalent twisting moment,
\[ T_e = \frac{\pi}{16} (d_{s1})^3 \tau \]  
...(ii)

From equations (i) and (ii), the diameter of the shaft at the junction of the crank \( (d_{s1}) \) may be determined.

(c) Design of shaft under the flywheel

Let \( d_s \) = Diameter of shaft under the flywheel.

The resultant bending moment \( (M_R) \) acting on the shaft is obtained in the similar way as discussed for dead centre position.

We know that horizontal bending moment acting on the shaft due to piston gas load,
\[ M_1 = F_p (a + b_2) - \left[ \sqrt{(H_{R1})^2 + (H_{T1})^2} \right] b_2 \]

and horizontal bending moment at the flywheel location due to belt pull,
\[ M_2 = H_1' \cdot b_2 = H_2' \cdot b_1 = \frac{(T_1 + T_2) b_1 b_2}{b} \]

\[ \therefore \text{Total horizontal bending moment,} \]
\[ M_H = M_1 + M_2 \]

Vertical bending moment due to the flywheel weight,
\[ M_V = V_1 \cdot b_2 = V_2 \cdot b_1 = \frac{W b_1 b_2}{b} \]

\[ \therefore \text{Resultant bending moment,} \]
\[ M_R = \sqrt{(M_H)^2 + (M_V)^2} \]

We know that twisting moment on the shaft,
\[ T = F_T \times r \]

\[ \therefore \text{Equivalent twisting moment,} \]
\[ T_e = \sqrt{(M_R)^2 + T^2} \]  
...(i)

We also know that equivalent twisting moment,
\[ T_e = \frac{\pi}{16} (d_s)^3 \tau \]  
...(ii)

From equations (i) and (ii), the diameter of shaft under the flywheel \( (d_s) \) may be determined.

Example 32.4. Design a plain carbon steel centre crankshaft for a single acting four stroke single cylinder engine for the following data:
Bore = 400 mm; Stroke = 600 mm; Engine speed = 200 r.p.m.; Mean effective pressure = 0.5 N/mm²; Maximum combustion pressure = 2.5 N/mm²; Weight of flywheel used as a pulley = 50 kN; Total belt pull = 6.5 kN.

When the crank has turned through 35° from the top dead centre, the pressure on the piston is 1N/mm² and the torque on the crank is maximum. The ratio of the connecting rod length to the crank radius is 5. Assume any other data required for the design.

Solution. Given: \( D = 400 \text{ mm} \); \( L = 600 \text{ mm} \) or \( r = 300 \text{ mm} \); \( p_m = 0.5 \text{ N/mm}^2 \); \( p = 2.5 \text{ N/mm}^2 \); \( W = 50 \text{ kN} \); \( T_1 + T_2 = 6.5 \text{ kN} \); \( \theta = 35^\circ \); \( p' = 1 \text{ N/mm}^2 \); \( l/r = 5 \)

We shall design the crankshaft for the two positions of the crank, i.e. firstly when the crank is at the dead centre; and secondly when the crank is at an angle of maximum twisting moment.

1. Design of the crankshaft when the crank is at the dead centre (See Fig. 32.18)

We know that the piston gas load,

\[
F_p = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} (400)^2 \times 2.5 = 314.2 \text{ kN}
\]

Assume that the distance \( b \) between the bearings 1 and 2 is equal to twice the piston diameter \( D \).

\[
\therefore \quad b = 2D = 2 \times 400 = 800 \text{ mm}
\]
and \[ b_1 = b_2 = \frac{b}{2} = \frac{800}{2} = 400 \text{ mm} \]

We know that due to the piston gas load, there will be two horizontal reactions \( H_1 \) and \( H_2 \) at bearings 1 and 2 respectively, such that
\[
H_1 = \frac{F_p \times b_1}{b} = \frac{314.2 \times 400}{800} = 157.1 \text{ kN}
\]
and
\[
H_2 = \frac{F_p \times b_2}{b} = \frac{314.2 \times 400}{800} = 157.1 \text{ kN}
\]

Assume that the length of the main bearings to be equal, \( i.e., c_1 = c_2 = c/2 \). We know that due to the weight of the flywheel acting downwards, there will be two vertical reactions \( V_2 \) and \( V_3 \) at bearings 2 and 3 respectively, such that
\[
V_2 = \frac{W \times c_1}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25 \text{ kN}
\]
and
\[
V_3 = \frac{W \times c_2}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25 \text{ kN}
\]

Due to the resultant belt tension \( (T_1 + T_2) \) acting horizontally, there will be two horizontal reactions \( H_2' \) and \( H_3' \) respectively, such that
\[
H_2' = \frac{(T_1 + T_2) \times c_1}{c} = \frac{(T_1 + T_2) \times c/2}{c} = \frac{6.5}{2} = 3.25 \text{ kN}
\]
and
\[
H_3' = \frac{(T_1 + T_2) \times c_2}{c} = \frac{(T_1 + T_2) \times c/2}{c} = \frac{6.5}{2} = 3.25 \text{ kN}
\]

Now the various parts of the crankshaft are designed as discussed below:

**(a) Design of crankpin**

Let \( d_c \) = Diameter of the crankpin in mm ; \\
\( l_c \) = Length of the crankpin in mm ; and \\
\( \sigma_b \) = Allowable bending stress for the crankpin. It may be assumed as 75 MPa or N/mm\(^2\).

We know that the bending moment at the centre of the crankpin,
\[
M_C = H_1 \cdot b_2 = 157.1 \times 400 = 62840 \text{ kN-mm} \quad \text{...(i)}
\]

We also know that
\[
M_C = \frac{\pi}{32} (d_c)^3 \sigma_b = \frac{\pi}{32} (d_c)^3 75 = 7.364 (d_c)^3 \text{ N-mm}
\]

Equating equations (i) and (ii), we have
\[
(d_c)^3 = \frac{62840}{7.364 \times 10^{-3}} = 8.53 \times 10^6
\]
or
\[ d_c = 204.35 \text{ say 205 mm Ans.} \]

We know that length of the crankpin,
\[
l_c = \frac{F_p}{d_c \cdot p_b} = \frac{314.2 \times 10^3}{205 \times 10} = 153.3 \text{ say 155 mm Ans.} \quad \text{...(Taking } p_b = 10 \text{ N/mm}\(^2\)}

**(b) Design of left hand crank web**

We know that thickness of the crank web,
\[
t = 0.65 d_c + 6.35 \text{ mm}
\]
\[
= 0.65 \times 205 + 6.35 = 139.6 \text{ say 140 mm Ans.} \]
and width of the crank web, \( w = 1.125 \times d_c + 12.7 \) mm
\[ = 1.125 \times 205 + 12.7 = 243.3 \text{ say 245 mm} \text{ Ans.} \]

We know that maximum bending moment on the crank web,
\[ M = H_1 \left( b_2 - \frac{L_c}{2} - \frac{L_t}{2} \right) \]
\[ = 157.1 \left( 400 - \frac{155}{2} - \frac{140}{2} \right) = 39 \, 668 \text{ kN-mm} \]

Section modulus, \( Z = \frac{1}{6} \times w. t^2 = \frac{1}{6} \times 245 \times (140)^2 = 800 \times 10^3 \text{ mm}^3 \)

\[ \therefore \text{ Bending stress, } \sigma_b = \frac{M}{Z} = \frac{39 \, 668}{800 \times 10^3} = 49.6 \times 10^{-3} \text{ kN/mm}^2 = 49.6 \text{ N/mm}^2 \]

We know that direct compressive stress on the crank web,
\[ \sigma_c = \frac{H_1}{w. t} = \frac{157.1}{245 \times 140} = 4.58 \times 10^{-3} \text{ kN/mm}^2 = 4.58 \text{ N/mm}^2 \]

\[ \therefore \text{ Total stress on the crank web} \]
\[ = \sigma_b + \sigma_c = 49.6 + 4.58 = 54.18 \text{ N/mm}^2 \text{ or MPa} \]

Since the total stress on the crank web is less than the allowable bending stress of 75 MPa, therefore, the design of the left hand crank web is safe.

(c) **Design of right hand crank web**

From the balancing point of view, the dimensions of the right hand crank web (i.e. thickness and width) are made equal to the dimensions of the left hand crank web.

(d) **Design of shaft under the flywheel**

Let \( d_s \) = Diameter of the shaft in mm.

Since the lengths of the main bearings are equal, therefore
\[ l_1 = l_2 = l_3 = 2 \left( \frac{b}{2} - \frac{l_c}{2} - \frac{l_t}{2} \right) = 2 \left( 400 - \frac{155}{2} - \frac{140}{2} \right) = 365 \text{ mm} \]

Assuming width of the flywheel as 300 mm, we have
\[ c = 365 + 300 = 665 \text{ mm} \]

*Hydrostatic transmission inside a tractor engine*
Allowing space for gearing and clearance, let us take \( c = 800 \text{ mm} \).

\[ c_1 = c_2 = \frac{c}{2} = \frac{800}{2} = 400 \text{ mm} \]

We know that bending moment due to the weight of flywheel,

\[ M_W = V_3 \cdot c_1 = 25 \times 400 = 10,000 \text{ kN-mm} = 10 \times 10^6 \text{ N-mm} \]

and bending moment due to the belt pull,

\[ M_T = H_f' \cdot c_1 = 3.25 \times 400 = 1300 \text{ kN-mm} = 1.3 \times 10^6 \text{ N-mm} \]

\[ \therefore \text{Resultant bending moment on the shaft,} \]

\[ M_S = \sqrt{(M_W)^2 + (M_T)^2} = \sqrt{(10 \times 10^6)^2 + (1.3 \times 10^6)^2} \]

\[ = 10.08 \times 10^6 \text{ N-mm} \]

We also know that bending moment on the shaft \((M_S)\),

\[ 10.08 \times 10^6 = \frac{\pi}{32} (d_f')^3 \sigma_b = \frac{\pi}{32} \left(\frac{d_f}{100}\right)^3 42 = 4.12 \left(\frac{d_f}{100}\right)^3 \]

\[ \therefore \left(\frac{d_f}{100}\right)^3 = 10.08 \times 10^6 / 4.12 = 2.45 \times 10^6 \text{ or } d_f = 134.7 \text{ say } 135 \text{ mm Ans.} \]

2. Design of the crankshaft when the crank is at an angle of maximum twisting moment

We know that piston gas load,

\[ F_p = \frac{\pi}{4} \times D^2 \times p' = \frac{\pi}{4} (400)^2 1 = 125,680 \text{ N} = 125.68 \text{ kN} \]

In order to find the thrust in the connecting rod \((F_Q)\), we should first find out the angle of inclination of the connecting rod with the line of stroke \((\text{i.e. angle } \phi)\). We know that

\[ \sin \phi = \frac{\sin \theta}{I/r} = \frac{\sin 35^\circ}{5} = 0.1147 \]

\[ \therefore \phi = \sin^{-1} (0.1147) = 6.58^\circ \]

We know that thrust in the connecting rod,

\[ F_Q = \frac{F_p \cos \phi}{\cos 6.58^\circ} = \frac{125.68}{0.9934} = 126.5 \text{ kN} \]

Tangential force acting on the crankshaft,

\[ F_T = F_Q \sin (\theta + \phi) = 126.5 \sin (35^\circ + 6.58^\circ) = 84 \text{ kN} \]

and radial force, \( F_R = F_Q \cos (\theta + \phi) = 126.5 \cos (35^\circ + 6.58^\circ) = 94.6 \text{ kN} \)

Due to the tangential force \((F_T)\), there will be two reactions at bearings 1 and 2, such that

\[ H_{T1} = \frac{F_T \times b_1}{b} = \frac{84 \times 400}{800} = 42 \text{ kN} \]

and

\[ H_{T2} = \frac{F_T \times b_2}{b} = \frac{84 \times 400}{800} = 42 \text{ kN} \]

Due to the radial force \((F_R)\), there will be two reactions at bearings 1 and 2, such that

\[ H_{R1} = \frac{F_R \times b_1}{b} = \frac{94.6 \times 400}{800} = 47.3 \text{ kN} \]

\[ H_{R2} = \frac{F_R \times b_2}{b} = \frac{94.6 \times 400}{800} = 47.3 \text{ kN} \]

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let

\[ d_c = \text{Diameter of crankpin in mm} \]
We know that the bending moment at the centre of the crankpin, 
\[ M_C = H_{R1} \times b_2 = 47.3 \times 400 = 18920 \text{ kN-mm} \]
and twisting moment on the crankpin, 
\[ T_C = H_{T1} \times r = 42 \times 300 = 12600 \text{ kN-mm} \]
∴ Equivalent twisting moment on the crankpin, 
\[ T_e = \sqrt{(M_C)^2 + (T_C)^2} = \sqrt{(18920)^2 + (12600)^2} = 22740 \text{ kN-mm} = 22.74 \times 10^6 \text{ N-mm} \]
We know that equivalent twisting moment \( T_e \),
\[ 22.74 \times 10^6 = \frac{\pi}{16} (d_c)^3 \tau = \frac{\pi}{16} (d_c)^3 35 = 6.873 (d_c)^3 \]...(Taking \( \tau = 35 \text{ MPa or N/mm}^2 \))
∴ \( (d_c)^3 = 22.74 \times 10^6 / 6.873 = 3.3 \times 10^6 \) or \( d_c = 149 \text{ mm} \)
Since this value of crankpin diameter \( i.e. d_c = 149 \text{ mm} \) is less than the already calculated value of \( d_c = 205 \text{ mm} \), therefore, we shall take \( d_c = 205 \text{ mm} \). \textbf{Ans.}

\( (b) \) \textbf{Design of shaft under the flywheel}
Let \( d_s \) = Diameter of the shaft in mm.
The resulting bending moment on the shaft will be same as calculated earlier, \( i.e. \)
\[ M_S = 10.08 \times 10^6 \text{ N-mm} \]
and twisting moment on the shaft, 
\[ T_S = F_T \times r = 84 \times 300 = 25200 \text{ kN-mm} = 25.2 \times 10^6 \text{ N-mm} \]
∴ Equivalent twisting moment on shaft, 
\[ T_e = \sqrt{(M_S)^2 + (T_S)^2} = \sqrt{(10.08 \times 10^6)^2 + (25.2 \times 10^6)^2} = 27.14 \times 10^6 \text{ N-mm} \]
We know that equivalent twisting moment \( T_e \),
\[ 27.14 \times 10^6 = \frac{\pi}{16} (d_s)^3 \tau = \frac{\pi}{16} (135)^3 \tau = 483156 \tau \]
∴ \( \tau = 27.14 \times 10^6 / 483156 = 56.17 \text{ N/mm}^2 \)
From above, we see that by taking the already calculated value of \( d_s = 135 \text{ mm} \), the induced shear stress is more than the allowable shear stress of 31 to 42 MPa. Hence, the value of \( d_s \) is calculated by taking \( \tau = 35 \text{ MPa or N/mm}^2 \) in the above equation, \( i.e. \)
\[ 27.14 \times 10^6 = \frac{\pi}{16} (d_s)^3 35 = 6.873 (d_s)^3 \]
∴ \( (d_s)^3 = 27.14 \times 10^6 / 6.873 = 3.95 \times 10^6 \) or \( d_s = 158 \text{ say 160 mm} \) \textbf{Ans.}

\( (c) \) \textbf{Design of shaft at the juncture of right hand crank arm}
Let \( d_{s1} \) = Diameter of the shaft at the juncture of the right hand crank arm.
We know that the resultant force at the bearing 1,
\[ R_1 = \sqrt{(H_{T1})^2 + (H_{R1})^2} = \sqrt{(42)^2 + (47.3)^2} = 63.3 \text{ kN} \]
∴ Bending moment at the juncture of the right hand crank arm,
\[ M_{S1} = R_1 \left( b_2 + \frac{t}{2} + \frac{t}{2} \right) - F_Q \left( \frac{l}{2} + \frac{t}{2} \right) \]
Internal Combustion Engine Parts

\[
= 63.3 \left( 400 + \frac{155}{2} + \frac{140}{2} \right) - 126.5 \left( \frac{155}{2} + \frac{140}{2} \right)
\]
\[
= 34.7 \times 10^3 - 18.7 \times 10^3 = 16 \times 10^3 \text{ kN-mm} = 16 \times 10^6 \text{ N-mm}
\]

and twisting moment at the juncture of the right hand crank arm,
\[
T_{S1} = F_T \times r = 84 \times 300 = 25 200 \text{ kN-mm} = 25.2 \times 10^6 \text{ N-mm}
\]

\[
∴ \text{ Equivalent twisting moment at the juncture of the right hand crank arm,}
\]
\[
T_e = \sqrt{(M_{S1})^2 + (T_{S1})^2}
\]
\[
= \sqrt{(16 \times 10^6)^2 + (25.2 \times 10^6)^2} = 29.85 \times 10^6 \text{ N-mm}
\]

We know that equivalent twisting moment \((T_e)\),
\[
29.85 \times 10^6 = \frac{\pi}{16} (d_{s1})^3 \tau = \frac{\pi}{16} (d_{s1})^3 42 = 8.25 (d_{s1})^3
\]
\[
\text{(Taking } \tau = 42 \text{ MPa or N/mm}^2)\]
\[
∴ (d_{s1})^3 = 29.85 \times 10^6 / 8.25 = 3.62 \times 10^6 \text{ or } d_{s1} = 153.5 \text{ say 155 mm Ans.}
\]

\(d) \text{ Design of right hand crank web}

Let
\[
\sigma_{br} = \text{Bending stress in the radial direction}; \text{ and }
\sigma_{bt} = \text{Bending stress in the tangential direction}.
\]

We also know that bending moment due to the radial component of \(F_Q\),
\[
M_R = H_{R2} \left( b_1 - \frac{l}{2} - \frac{t}{2} \right) = 47.3 \left( 400 - \frac{155}{2} - \frac{140}{2} \right) \text{ kN-mm}
\]
\[
= 11.94 \times 10^3 \text{ kN-mm} = 11.94 \times 10^6 \text{ N-mm} \hspace{1cm} \ldots(i)
\]

We also know that bending moment,
\[
M_R = \sigma_{br} \times Z = \sigma_{br} \times \frac{1}{6} \times w J^2 \hspace{1cm} \ldots(,:) \text{ Z} = \frac{1}{6} \times w J^2
\]
\[
11.94 \times 10^6 = \sigma_{br} \times \frac{1}{6} \times 245 (140)^2 = 800 \times 10^3 \sigma_{br}
\]
\[
∴ \sigma_{br} = 11.94 \times 10^6 / 800 \times 10^3 = 14.9 \text{ N/mm}^2 \text{ or MPa}
\]

We know that bending moment due to the tangential component of \(F_Q\),
\[
M_T = F_T \left( r - \frac{d_{s1}}{2} \right) = 84 \left( 300 - \frac{155}{2} \right) = 18 690 \text{ kN-mm}
\]
\[
= 18.69 \times 10^6 \text{ N-mm}
\]

We also know that bending moment,
\[
M_T = \sigma_{bt} \times Z = \sigma_{bt} \times \frac{1}{6} \times t w^2 \hspace{1cm} \ldots(,:) \text{ Z} = \frac{1}{6} \times t w^2
\]
\[
18.69 \times 10^6 = \sigma_{bt} \times \frac{1}{6} \times 140(245)^2 = 1.4 \times 10^6 \sigma_{bt}
\]
\[
∴ \sigma_{bt} = 18.69 \times 10^6 / 1.4 \times 10^6 = 13.35 \text{ N/mm}^2 \text{ or MPa}
\]

Direct compressive stress,
\[
\sigma_b = \frac{F_R}{2w \cdot t} = \frac{94.6}{2 \times 245 \times 140} = 1.38 \times 10^{-3} \text{ kN/mm}^2 = 1.38 \text{ N/mm}^2
\]
and total compressive stress,
\[ \sigma_c = \sigma_{br} + \sigma_{bt} + \sigma_d = 14.9 + 13.35 + 1.38 = 29.63 \text{ N/mm}^2 \text{ or MPa} \]

We know that twisting moment on the arm,
\[ T = H_{T2} \left( \frac{b_1 - l_c}{2} \right) = 42 \left( \frac{400 - 155}{2} \right) = 13.545 \text{ kN-mm} \]
\[ = 13.545 \times 10^6 \text{ N-mm} \]

and shear stress on the arm,
\[ \tau = \frac{T}{Z_p} = \frac{4.5T}{w.T} = \frac{4.5 \times 13.545 \times 10^6}{245 (140)^2} = 12.7 \text{ N/mm}^2 \text{ or MPa} \]

We know that total or maximum combined stress,
\[ (\sigma_c)_{max} = \frac{\sigma_c}{2} + \frac{1}{2} \sqrt{(\sigma_c)^2 + 4\tau^2} \]
\[ = \frac{29.63}{2} + \frac{1}{2} \sqrt{(29.63)^2 + 4(12.7)^2} = 14.815 + 19.5 = 34.315 \text{ MPa} \]

Since the maximum combined stress is within the safe limits, therefore, the dimension \( w = 245 \text{ mm} \) is accepted.

(e) **Design of left hand crank web**

The dimensions for the left hand crank web may be made same as for right hand crank web.

( f ) **Design of crankshaft bearings**

Since the bearing 2 is the most heavily loaded, therefore, only this bearing should be checked for bearing pressure.

We know that the total reaction at bearing 2,
\[ R_2 = \frac{F_p}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2} = \frac{314.2}{2} + \frac{50}{2} + \frac{6.5}{2} = 185.35 \text{ kN} = 185350 \text{ N} \]

∴ Total bearing pressure
\[
\frac{R_2}{I_2 \cdot d_1} = \frac{185 \times 350}{365 \times 155} = 3.276 \text{ N/mm}^2
\]

Since this bearing pressure is less than the safe limit of 5 to 8 N/mm², therefore, the design is safe.

Example 32.5. **Design a side or overhung crankshaft for a 250 mm × 300 mm gas engine.** The weight of the flywheel is 30 kN and the explosion pressure is 2.1 N/mm². The gas pressure at the maximum torque is 0.9 N/mm², when the crank angle is 35° from I. D. C. The connecting rod is 4.5 times the crank radius.

**Solution.** Given : \(D = 250 \text{ mm} \); \(L = 300 \text{ mm} \) or \(r = L / 2 = 300 / 2 = 150 \text{ mm} \); \(W = 30 \text{ kN} = 30 \times 10^3 \text{ N} \); \(p = 2.1 \text{ N/mm}^2 \); \(p' = 0.9 \text{ N/mm}^2 \); \(l = 4.5 \text{ or } l / r = 4.5 \)

We shall design the crankshaft for the two positions of the crank, i.e. firstly when the crank is at the dead centre and secondly when the crank is at an angle of maximum twisting moment.

1. **Design of crankshaft when the crank is at the dead centre** (See Fig. 32.18)

We know that piston gas load,

\[
F_p = \frac{\pi}{4} \times D^2 \times p
\]

\[
= \frac{\pi}{4} (250)^2 \times 2.1 = 103 \times 10^3 \text{ N}
\]

Now the various parts of the crankshaft are designed as discussed below:

(a) **Design of crankpin**

Let \(d_c\) = Diameter of the crankpin in mm, and 
\(l_c = \) Length of the crankpin = 0.8 \(d_c\)  
...(Assume)

Considering the crankpin in bearing, we have 

\[
F_p = d_c \cdot l_c \cdot p_b
\]

\[
103 \times 10^3 = d_c \times 0.8 \times d_c \times 10 = 8 \times (d_c)^2
\]

...(Taking \(p_b = 10 \text{ N/mm}^2\))

\[
\therefore \ (d_c)^2 = 103 \times 10^3 / 8 = 12 \, 875 \ \text{or } d_c = 113.4 \ \text{say } 115 \text{ mm}
\]

and 
\(l_c = 0.8 \times d_c = 0.8 \times 115 = 92 \text{ mm}\)

Let us now check the induced bending stress in the crankpin.

We know that bending moment at the crankpin,

\[
M = \frac{3}{4} F_p \times l_c = \frac{3}{4} \times 103 \times 10^3 \times 92 = 7107 \times 10^3 \text{ N-mm}
\]

and section modulus of the crankpin,

\[
Z = \frac{\pi}{32} (d_c)^3 = \frac{\pi}{32} (115)^3 = 149 \times 10^3 \text{ mm}^3
\]

\[
\therefore \text{Bending stress induced } M / Z = \frac{7107 \times 10^3}{149 \times 10^3} = 47.7 \text{ N/mm}^2 \text{ or MPa}
\]

Since the induced bending stress is within the permissible limits of 60 MPa, therefore, design of crankpin is safe.
(b) Design of bearings

Let  \( d_1 \) = Diameter of the bearing 1.
Let us take thickness of the crank web, \( t = 0.6 \times 115 = 69 \) or \( 70 \) mm
and length of the bearing, \( l_1 = 1.7 \times 115 = 195.5 \) say 200 mm

We know that bending moment at the centre of the bearing 1,

\[
M = F_p (0.75l_1 + t + 0.5 l_1)
\]

\[
= 103 \times 10^3 (0.75 \times 92 + 70 + 0.5 \times 200) = 24.6 \times 10^6 \text{ N-mm}
\]

We also know that bending moment \( M \),

\[
24.6 \times 10^6 = \frac{\pi}{32} (d_1)^3 \sigma_b = \frac{\pi}{32} (d_1)^3 60 = 5.9 (d_1)^3
\]

\[
\therefore (d_1)^3 = 24.6 \times 10^6 / 5.9 = 4.2 \times 10^6 \text{ or } d_1 = 161.3 \text{ mm say 162 mm Ans.}
\]

(c) Design of crank web

Let  \( w \) = Width of the crank web in mm.

We know that bending moment on the crank web,

\[
M = F_p (0.75l_1 + t + 0.5 t)
\]

\[
= 103 \times 10^3 (0.75 \times 92 + 0.5 \times 70) = 10.7 \times 10^6 \text{ N-mm}
\]

and section modulus,

\[
Z = \frac{1}{6} \times w \times r^2 = \frac{1}{6} \times w(70)^2 = 817 w \text{ mm}^3
\]

\[
\therefore \text{Bending stress, } \sigma_b = \frac{M}{Z} = \frac{10.7 \times 10^6}{817 w} = \frac{13 \times 10^3}{w} \text{ N/mm}^2
\]

and direct compressive stress,

\[
\sigma_d = \frac{F_p}{w t} = \frac{103 \times 10^3}{w \times 70} = \frac{1.47 \times 10^3}{w} \text{ N/mm}^2
\]

We know that total stress on the crank web,

\[
\sigma_T = \sigma_b + \sigma_d = \frac{13 \times 10^3}{w} + \frac{1.47 \times 10^3}{w} = \frac{14.47 \times 10^3}{w} \text{ N/mm}^2
\]

The total stress should not exceed the permissible limit of 60 MPa or N/mm².

\[
\therefore 60 = \frac{14.47 \times 10^3}{w} \text{ or } w = \frac{14.47 \times 10^3}{60} = 241 \text{ say 245 mm Ans.}
\]

(d) Design of shaft under the flywheel.

Let  \( d_s \) = Diameter of shaft under the flywheel.

First of all, let us find the horizontal and vertical reactions at bearings 1 and 2. Assume that the width of flywheel is 250 mm and \( l_1 = l_2 = 200 \) mm.

Allowing for certain clearance, the distance

\[
b = 250 + \frac{l_1}{2} + \frac{l_2}{2} + \text{clearance}
\]

\[
= 250 + \frac{200}{2} + \frac{200}{2} + 20 = 470 \text{ mm}
\]

and

\[
a = 0.75 l_1 + t + 0.5 l_1
\]

\[
= 0.75 \times 92 + 70 + 0.5 \times 200 = 239 \text{ mm}
\]

We know that the horizontal reactions \( H_1 \) and \( H_2 \) at bearings 1 and 2, due to the piston gas load \( F_p \) are
\[ H_1 = \frac{F_p (a + b)}{b} = \frac{103 \times 10^3 (239 + 470)}{470} = 155.4 \times 10^3 \text{ N} \]

and

\[ H_2 = \frac{F_p \times a}{b} = \frac{103 \times 10^3 \times 239}{470} = 52.4 \times 10^3 \text{ N} \]

Assuming \( b_1 = b_2 = b/2 \), the vertical reactions \( V_1 \) and \( V_2 \) at bearings 1 and 2 due to the weight of the flywheel are

\[ V_1 = \frac{W \cdot b_1}{b} = \frac{W \times b/2}{b} = \frac{W}{2} = \frac{30 \times 10^3}{2} = 15 \times 10^3 \text{ N} \]

and

\[ V_2 = \frac{W \cdot b_2}{b} = \frac{W \times b/2}{b} = \frac{W}{2} = \frac{30 \times 10^3}{2} = 15 \times 10^3 \text{ N} \]

Since there is no belt tension, therefore the horizontal reactions due to the belt tension are neglected.

We know that horizontal bending moment at the flywheel location due to piston gas load.

\[ M_1 = F_p (a + b) - H_1 \cdot b_2 \]

\[ = 103 \times 10^3 \left( \frac{239 + 470}{2} \right) - 155.4 \times 10^3 \times \frac{470}{2} \]

\[ = 48.8 \times 10^6 - 36.5 \times 10^6 = 12.3 \times 10^6 \text{ N-mm} \]

Since there is no belt pull, therefore, there will be no horizontal bending moment due to the belt pull, \( i.e. M_2 = 0 \).

\[ \therefore \text{Total horizontal bending moment,} \]

\[ M_{HH} = M_1 + M_2 = M_1 = 12.3 \times 10^6 \text{ N-mm} \]

We know that vertical bending moment due to the flywheel weight,

\[ M_V = \frac{W \cdot b_1 \cdot b_2}{b} = \frac{W \times b/2 \times b}{2 \times 2 \times b} = \frac{W \times b}{4} \]

\[ = \frac{30 \times 10^3 \times 470}{4} = 3.525 \times 10^6 \text{ N-mm} \]
Resultant bending moment,
\[ M_R = \sqrt{(M_H)^2 + (M_V)^2} = \sqrt{(12.3 \times 10^6)^2 + (3.525 \times 10^6)^2} \]
\[ = 12.8 \times 10^6 \text{ N-mm} \]

We know that bending moment \( M_R \),
\[ 12.8 \times 10^6 = \frac{\pi}{32} (d_s)^3 \sigma_b = \frac{\pi}{32} (d_s)^3 60 = 5.9(d_s)^3 \]
\[ \therefore (d_s)^3 = 12.8 \times 10^6 / 5.9 = 2.17 \times 10^6 \text{ or } d_s = 129 \text{ mm} \]

Actually \( d_s \) should be more than \( d_1 \). Therefore let us take
\[ d_s = 200 \text{ mm } \text{ Ans.} \]

2. Design of crankshaft when the crank is at an angle of maximum twisting moment

We know that piston gas load,
\[ F_p = \frac{\pi}{4} \times D^2 \times p' = \frac{\pi}{4}(250)^2 \times 0.9 = 44200 \text{ N} \]

In order to find the thrust in the connecting rod \( (F_Q) \), we should first find out the angle of inclination of the connecting rod with the line of stoke \((i.e.\ angle \phi)\). We know that
\[ \sin \phi = \frac{\sin \theta}{I/R} = \frac{\sin 35^\circ}{4.5} = 0.1275 \]
\[ \therefore \phi = \sin^{-1}(0.1275) = 7.32^\circ \]

We know that thrust in the connecting rod,
\[ F_Q = \frac{F_p}{\cos \phi} = \frac{44200}{\cos 7.32^\circ} = 44565 \text{ N} \]

Tangential force acting on the crankshaft,
\[ F_T = F_Q \sin (\theta + \phi) = 44565 \sin (35^\circ + 7.32^\circ) = 30 \times 10^3 \text{ N} \]

and radial force,
\[ F_R = F_Q \cos (\theta + \phi) = 44565 \cos (35^\circ + 7.32^\circ) = 33 \times 10^3 \text{ N} \]

Due to the tangential force \( (F_T) \), there will be two reactions at the bearings 1 and 2, such that
\[ H_{T1} = \frac{F_T(a + b)}{b} = \frac{30 \times 10^3 (239 + 470)}{470} = 45 \times 10^3 \text{ N} \]
and
\[ H_{T2} = \frac{F_T \times a}{b} = \frac{30 \times 10^3 \times 239}{470} = 15.3 \times 10^3 \text{ N} \]

Due to the radial force \( (F_R) \), there will be two reactions at the bearings 1 and 2, such that
\[ H_{R1} = \frac{F_R(a + b)}{b} = \frac{33 \times 10^3 (239 + 470)}{470} = 49.8 \times 10^3 \text{ N} \]
and
\[ H_{R2} = \frac{F_R \times a}{b} = \frac{33 \times 10^3 \times 239}{470} = 16.8 \times 10^3 \text{ N} \]

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crank web

We know that bending moment due to the tangential force,
\[ M_{bt} = F_T \left( r - \frac{d_s}{2} \right) = 30 \times 10^3 \left( 150 - \frac{180}{2} \right) = 1.8 \times 10^6 \text{ N-mm} \]
Bending stress due to the tangential force,

$$\sigma_{bt} = \frac{M_{bt}}{Z} = \frac{6M_{bt}}{I.w^3} = \frac{6 \times 1.8 \times 10^6}{70 (245)^2} \quad \text{... (} Z = \frac{1}{6} \times \frac{1.8 \times 10^6}{70 (245)^2} \text{)}$$

Bending moment due to the radial force,

$$M_{br} = F_R (0.75 l + 0.5 t)$$

$$= 33 \times 10^3 (0.75 \times 92 + 0.5 \times 70) = 3.43 \times 10^6 \text{ N-mm}$$

Bending stress due to the radial force,

$$\sigma_{br} = \frac{M_{br}}{Z} = \frac{6 M_{br}}{I.w^2} \quad \text{... (} Z = \frac{1}{6} \times \frac{6 M_{br}}{I.w^2} \text{)}$$

$$= \frac{6 \times 3.43 \times 10^6}{245 (70)^2} = 17.1 \text{ N/mm}^2 \text{ or MPa}$$

Schematic of a 4 cylinder IC engine
We know that direct compressive stress,
\[ \sigma_d = \frac{F_R}{w \cdot t} = \frac{33 \times 10^3}{245 \times 70} = 1.9 \text{ N/mm}^2 \text{ or MPa} \]

\[ \therefore \] Total compressive stress,
\[ \sigma_c = \sigma_{by} + \sigma_{br} + \sigma_d = 2.6 + 17.1 + 1.9 = 21.6 \text{ MPa} \]

We know that twisting moment due to the tangential force,
\[ T = F_T (0.75 l_c + 0.5 t) \]
\[ = 30 \times 10^3 (0.75 \times 92 + 0.5 \times 70) = 3.12 \times 10^6 \text{ N-mm} \]

\[ \therefore \] Shear stress,
\[ \tau = \frac{T}{Z_p} = \frac{4.5 T}{w \cdot t^2} = \frac{4.5 \times 3.12 \times 10^6}{245 (70)^2} \]
\[ = 11.7 \text{ N/mm}^2 \text{ or MPa} \]

We know that total or maximum stress,
\[ \sigma_{max} = 2 \frac{T}{w \cdot t^2} + \frac{1}{2} \sqrt{\left(\sigma_c + \tau \right)^2 + 4 \tau^2} = 21.6 + \frac{1}{2} \sqrt{(21.6)^2 + 4(11.7)^2} \]
\[ = 26.7 \text{ MPa} \]

Since this stress is less than the permissible value of 60 MPa, therefore, the design is safe.

\( b \) Design of shaft at the junction of crank

Let \( d_{s1} \) = Diameter of shaft at the junction of crank.

We know that bending moment at the junction of crank,
\[ M = F_Q (0.75 l_c + t) = 44 \times 565 (0.75 \times 92 + 70) = 6.2 \times 10^6 \text{ N-mm} \]

and twisting moment,
\[ T = F_T \times r = 30 \times 10^3 \times 150 = 4.5 \times 10^6 \text{ N-mm} \]

\[ \therefore \] Equivalent twisting moment,
\[ T_e = \sqrt{M^2 + T^2} = \sqrt{(6.2 \times 10^6)^2 + (4.5 \times 10^6)^2} = 7.66 \times 10^6 \text{ N-mm} \]

We also know that equivalent twisting moment \( (T_e) \),
\[ 7.66 \times 10^6 = \frac{\pi}{16} (d_{s1})^3 \tau = \frac{\pi}{16} (180^3) \tau = 1.14 \times 10^6 \tau \]
...(Taking \( d_{s1} = d_1 \))
\[ \therefore \] \( \tau = 7.66 \times 10^6 / 1.14 \times 10^6 = 6.72 \text{ N/mm}^2 \text{ or MPa} \)

Since the induced shear stress is less than the permissible limit of 30 to 40 MPa, therefore, the design is safe.

\( c \) Design of shaft under the flywheel

Let \( d_s \) = Diameter of shaft under the flywheel.

We know that horizontal bending moment acting on the shaft due to piston gas load,
\[ M_H = F_p (a + b_2) - \left[ \sqrt{(H_{R1})^2 + (H_{T1})^2} \right] b_2 \]
\[ = 44 \times 200 \left[ 239 + \frac{470}{2} \right] - \left[ \sqrt{(49.8 \times 10^3)^2 + (45 \times 10^3)^2} \right] \frac{470}{2} \]
\[ = 20.95 \times 10^6 - 15.77 \times 10^6 = 5.18 \times 10^6 \text{ N-mm} \]

and bending moment due to the flywheel weight...
Internal Combustion Engine Parts

\[ M_v = \frac{W_b \cdot b_2}{b} = \frac{30 \times 10^3 \times 235 \times 235}{470} = 3.53 \times 10^6 \text{ N-mm} \]

\[ (b_1 = b_2 = b / 2 = 470 / 2 = 235 \text{ mm}) \]

.: Resultant bending moment,
\[ M_R = \sqrt{(M_1)^2 + (M_v)^2} = \sqrt{(5.18 \times 10^6)^2 + (3.53 \times 10^6)^2} \]
\[ = 6.27 \times 10^6 \text{ N-mm} \]

We know that twisting moment on the shaft,
\[ T = F_r \times r = 30 \times 10^3 \times 150 = 4.5 \times 10^6 \text{ N-mm} \]

.: Equivalent twisting moment,
\[ T_e = \sqrt{(M_R)^2 + T^2} = \sqrt{(6.27 \times 10^6)^2 + (4.5 \times 10^6)^2} \]
\[ = 7.72 \times 10^6 \text{ N-mm} \]

We also know that equivalent twisting moment \( (T_e) \),
\[ 7.72 \times 10^6 = \frac{\pi}{16} (d_s)^3 \tau = \frac{\pi}{16} (d_s)^3 30 = 5.9 (d_s)^3 \]
\[ \text{...(Taking } \tau = 30 \text{ MPa}) \]

\[ (d_s)^3 = 7.72 \times 10^6 / 5.9 = 1.31 \times 10^6 \text{ or } d_s = 109 \text{ mm} \]

Actually, \( d_s \) should be more than \( d_1 \). Therefore let us take \( d_s = 200 \text{ mm} \)

**32.22 Valve Gear Mechanism**

The valve gear mechanism of an I.C. engine consists of those parts which actuate the inlet and exhaust valves at the required time with respect to the position of piston and crankshaft. Fig. 32.20 (a) shows the valve gear arrangement for vertical engines. The main components of the mechanism are valves, rocker arm, * valve springs, ** push rod, *** cam and camshaft.

---

* For the design of springs, refer Chapter 23.
** For the design of push rod, refer Chapter 16 (Art. 16.14).
*** For the design of cams, refer to Authors’ popular book on ‘Theory of Machines’.

---

![Fig. 32.20. Valve gear mechanism.](attachment:valve_gear_mechanism.png)
The fuel is admitted to the engine by the inlet valve and the burnt gases are escaped through the
exhaust valve. In vertical engines, the cam moving on the rotating camshaft pushes the cam follower
and push rod upwards, thereby transmitting the cam action to rocker arm. The camshaft is rotated by
the toothed belt from the crankshaft. The rocker arm is pivoted at its centre by a fulcrum pin. When
one end of the rocker arm is pushed up by the push rod, the other end moves downward. This pushes
down the valve stem causing the valve to move down, thereby opening the port. When the cam
follower moves over the circular portion of cam, the pushing action of the rocker arm on the valve is
released and the valve returns to its seat and closes it by the action of the valve spring.

In some of the modern engines, the camshaft is located at cylinder head level. In such cases, the
push rod is eliminated and the roller type cam follower is made part of the rocker arm. Such an
arrangement for the horizontal engines is shown in Fig. 32.20 (b).

32.23 Valves

The valves used in internal combustion engines are
of the following three types :

1. Poppet or mushroom valve ; 2. Sleeve valve ;
3. Rotary valve.

Out of these three valves, poppet valve, as shown
in Fig. 32.21, is very frequently used. It consists of head,
face and stem. The head and face of the valve is sepa-
rated by a small margin, to avoid sharp edge of the valve
and also to provide provision for the regrinding of the
face. The face angle generally varies from 30° to 45°.
The lower part of the stem is provided with a groove in
which spring retainer lock is installed.

Since both the inlet and exhaust valves are subjected
to high temperatures of 1930°C to 2200°C during the
power stroke, therefore, it is necessary that the material
of the valves should withstand these temperatures. Thus
the material of the valves must have good heat conduction,
heat resistance, corrosion resistance, wear resistance and
shock resistance. It may be noted that the temperature at
the inlet valve is less as compared to exhaust valve. Thus,
the inlet valve is generally made of nickel chromium alloy
steel and the exhaust valve (which is subjected to very high
temperature of exhaust gases) is made from silchrome steel
which is a special alloy of silicon and chromium.

In designing a valve, it is required to determine the following dimensions:

(a) Size of the valve port

Let

\[ a_p = \text{Area of the port}, \]
\[ v_p = \text{Mean velocity of gas flowing through the port}, \]
\[ a = \text{Area of the piston}, \] and
\[ v = \text{Mean velocity of the piston}. \]

We know that

\[ a_p v_p = a v \]

∴

\[ a_p = \frac{a v}{v_p} \]
The mean velocity of the gas ($v_p$) may be taken from the following table.

**Table 32.3. Mean velocity of the gas ($v_p$)**

<table>
<thead>
<tr>
<th>Type of engine</th>
<th>Mean velocity of the gas ($v_p$) m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inlet valve</td>
</tr>
<tr>
<td>Low speed</td>
<td>33 – 40</td>
</tr>
<tr>
<td>High speed</td>
<td>80 – 90</td>
</tr>
</tbody>
</table>

Sometimes, inlet port is made 20 to 40 percent larger than exhaust port for better cylinder charging.

**b) Thickness of the valve disc**

The thickness of the valve disc ($t$), as shown in Fig. 32.22, may be determined empirically from the following relation, i.e.

$$t = k.d_p \sqrt{\frac{p}{\sigma_b}}$$

where

- $k$ = Constant = 0.42 for steel and 0.54 for cast iron,
- $d_p$ = Diameter of the port in mm,
- $p$ = Maximum gas pressure in N/mm$^2$, and
- $\sigma_b$ = Permissible bending stress in MPa or N/mm$^2$ = 50 to 60 MPa for carbon steel and 100 to 120 MPa for alloy steel.

**c) Maximum lift of the valve**

$h$ = Lift of the valve.

The lift of the valve may be obtained by equating the area across the valve seat to the area of the port. For a conical valve, as shown in Fig. 32.22, we have

$$\pi d_p h \cos \alpha = \frac{\pi}{4} (d_p)^2$$

or

$$h = \frac{d_p}{4 \cos \alpha}$$

where

$\alpha$ = Angle at which the valve seat is tapered = 30° to 45°.
In case of flat headed valve, the lift of valve is given by

\[ h = \frac{d_p}{4} \]

...(In this case, \( \alpha = 0° \))

The valve seats usually have the same angle as the valve seating surface. But it is preferable to make the angle of valve seat \( \frac{1}{2}° \) to \( 1° \) larger than the valve angle as shown in Fig. 32.23. This results in more effective seat.

(d) Valve stem diameter

The valve stem diameter \((d_s)\) is given by

\[ d_s = \frac{d_p}{8} + 6.35 \text{ mm to } \frac{d_p}{8} + 11 \text{ mm} \]

Note: The valve is subjected to spring force which is taken as concentrated load at the centre. Due to this spring force \((F_s)\), the stress in the valve \((\sigma_t)\) is given by

\[ \sigma_t = \frac{1.4 F_s t}{r^2} \left( 1 - \frac{2d_p}{3d_s} \right) \]

Example 32.6. The conical valve of an I.C. engine is 60 mm in diameter and is subjected to a maximum gas pressure of 4 N/mm². The safe stress in bending for the valve material is 46 MPa. The valve is made of steel for which \( k = 0.42 \). The angle at which the valve disc seat is tapered is 30°.

Determine : 1. thickness of the valve head ; 2. stem diameter ; and 3. maximum lift of the valve.

Solution. Given : \( d_p = 60 \text{ mm} ; p = 4 \text{ N/mm}^2 ; \sigma_b = 46 \text{ MPa} = 46 \text{ N/mm}^2 ; k = 0.42 ; \alpha = 30° \)

1. Thickness of the valve head

We know that thickness of the valve head,

\[ t = k \cdot d_p \sqrt{\frac{p}{\sigma_b}} = 0.42 \times 60 \sqrt{\frac{4}{46}} = 7.43 \text{ say } 7.5 \text{ mm Ans.} \]

2. Stem diameter

We know that stem diameter,

\[ d_s = \frac{d_p}{8} + 6.35 = \frac{60}{8} + 6.35 = 13.85 \text{ say } 14 \text{ mm Ans.} \]

3. Maximum lift of the valve

We know that maximum lift of the valve,

\[ h = \frac{d_p}{4 \cos \alpha} = \frac{60}{4 \cos 30°} = \frac{60}{4 \times 0.866} = 17.32 \text{ say } 17.4 \text{ mm Ans.} \]

32.24 Rocker Arm

The *rocker arm is used to actuate the inlet and exhaust valves motion as directed by the cam and follower. It may be made of cast iron, cast steel, or malleable iron. In order to reduce inertia of the rocker arm, an I-section is used for the high speed engines and it may be rectangular section for low speed engines. In four stroke engines, the rocker arms for the exhaust valve is the most heavily loaded. Though the force required to operate the inlet valve is relatively small, yet it is usual practice to make the rocker

* The rocker arm has also been discussed in Chapter 15 on Levers (Refer Art. 15.9).
arm for the inlet valve of the same dimensions as that for exhaust valve. A typical rocker arm for operating the exhaust valve is shown in Fig. 32.24. The lever ratio $a / b$ is generally decided by considering the space available for rocker arm. For moderate and low speed engines, $a / b$ is equal to one. For high speed engines, the ratio $a / b$ is taken as $1 / 1.3$. The various forces acting on the rocker arm of exhaust valve are the gas load, spring force and force due to valve acceleration.

![Fig. 32.24. Rocker arm for exhaust valve.](image)

Let

- $m_v = \text{Mass of the valve},$
- $d_v = \text{Diameter of the valve head},$
- $h = \text{Lift of the valve},$
- $a = \text{Acceleration of the valve},$
- $p_c = \text{Cylinder pressure or back pressure when the exhaust valve opens,}$
- $p_s = \text{Maximum suction pressure}.$

We know that gas load,

$$P = \text{Area of valve} \times \text{Cylinder pressure when the exhaust valve opens}$$

$$= \frac{\pi}{4} (d_v)^2 p_c$$

Spring force,

$$F_s = \text{Area of valve} \times \text{Maximum suction pressure}$$

$$= \frac{\pi}{4} (d_v)^2 p_s$$

and force due to valve acceleration,

$$F_{va} = \text{Mass of valve} \times \text{Acceleration of valve}$$

$$= m_v \times a$$

∴ Maximum load on the rocker arm for exhaust valve,

$$F_e = P + F_s + F_{va}$$

It may be noted that maximum load on the rocker arm for inlet valve is

$$F_i = F_s + F_{va}$$

Since the maximum load on the rocker arm for exhaust valve is more than that of inlet valve, therefore, the rocker arm must be designed on the basis of maximum load on the rocker arm for exhaust valve, as discussed below:

1. **Design for fulcrum pin.** The load acting on the fulcrum pin is the total reaction ($R_p$) at the fulcrum point.
Let \( d_1 \) = Diameter of the fulcrum pin, and 
\( l_1 \) = Length of the fulcrum pin.

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin,
\[
R_f = d_1 \cdot l_1 \cdot p_b
\]

The ratio of \( l_1 / d_1 \) is taken as 1.25 and the bearing pressure \( (p_b) \) for ordinary lubrication is taken from 3.5 to 6 N/mm\(^2\) and it may go up to 10.5 N/mm\(^2\) for forced lubrication.

The pin should be checked for the induced shear stress.

The thickness of the phosphor bronze bush may be taken from 2 to 4 mm. The outside diameter of the boss at the fulcrum is usually taken twice the diameter of the fulcrum pin.

2. Design for forked end. The forked end of the rocker arm carries a roller by means of a pin. For uniform wear, the roller should revolve in the eyes. The load acting on the roller pin is \( F_c \).

Let \( d_2 \) = Diameter of the roller pin, and 
\( l_2 \) = Length of the roller pin.

Considering the bearing of the roller pin. We know that load on the roller pin,
\[
F_c = d_2 \cdot l_2 \cdot p_b
\]

The ratio of \( l_2 / d_2 \) may be taken as 1.25. The roller pin should be checked for induced shear stress.

The roller pin is fixed in the eye and the thickness of each eye is taken as half the length of the roller pin.

\[
\therefore \text{Thickness of each eye} = \frac{l_2}{2}
\]

The radial thickness of eye \( (t_3) \) is taken as \( d_1 / 2 \). Therefore overall diameter of the eye,
\[
D_1 = 2 \cdot d_1
\]

The outer diameter of the roller is taken slightly larger (at least 3 mm more) than the outer diameter of the eye.

A clearance of 1.5 mm between the roller and the fork on either side of the roller is provided.

3. Design for rocker arm cross-section. The rocker arm may be treated as a simply supported beam and loaded at the fulcrum point. We have already discussed that the rocker arm is generally of I-section but for low speed engines, it can be of rectangular section. Due to the load on the valve, the rocker arm is subjected to bending moment.

Let \( l \) = Effective length of each rocker arm, and 
\( \sigma_b \) = Permissible bending stress.

We know that bending moment on the rocker arm,
\[
M = F_c \times l \quad \ldots \,(i)
\]

We also know that bending moment,
\[
M = \sigma_b \times Z \quad \ldots \,(ii)
\]

where 
\( Z \) = Section modulus.

From equations \((i)\) and \((ii)\), the value of \( Z \) is obtained and thus the dimensions of the section are determined.

4. Design for tappet. The tappet end of the rocker arm is made circular to receive the tappet which is a stud with a lock nut. The compressive load acting on the tappet is the maximum load on the rocker arm for the exhaust valve \( (F_e) \).

Let \( d_c \) = Core diameter of the tappet, and 
\( \sigma_c \) = Permissible compressive stress for the material of the tappet which is made of mild steel. It may be taken as 50 MPa.

We know that load on the tappet,
\[
F_e = \frac{\pi}{4} (d_c)^2 \sigma_c
\]

From this expression, the core diameter of the tappet is determined. The outer or nominal diameter of the tappet \( (d_n) \) is given as
The diameter of the circular end of the rocker arm ($D_3$) and its depth ($t_4$) is taken as twice the nominal diameter of the tappet ($d_n$), i.e.,

$$D_3 = 2 \times d_n \quad \text{and} \quad t_4 = 2 \times d_n$$

5. Design for valve spring. The valve spring is used to provide sufficient force during the valve lifting process in order to overcome the inertia of valve gear and to keep it with the cam without bouncing. The spring is generally made from plain carbon spring steel. The total load for which the spring is designed is equal to the sum of initial load and load at full lift.

Let

- $W_1 = \text{Initial load on the spring}$
- $W_2 = \text{Load at full lift}$

\[ W = W_1 + W_2 \]

\[ W = W_1 + W_2 \]

Note: Here we are only interested in calculating the total load on the spring. The design of the valve spring is done in the similar ways as discussed for compression springs in Chapter 23 on Springs.

Example 32.7. Design a rocker arm, and its bearings, tappet, roller and valve spring for the exhaust valve of a four stroke I.C. engine from the following data:

- Diameter of the valve head = 80 mm; Lift of the valve = 25 mm; Mass of associated parts with the valve = 0.4 kg; Angle of action of camshaft = 110°; R. P. M. of the crankshaft = 1500.
- From the probable indicator diagram, it has been observed that the greatest back pressure when the exhaust valve opens is 0.4 N/mm² and the greatest suction pressure is 0.02 N/mm² below atmosphere.

The rocker arm is to be of I-section and the effective length of each arm may be taken as 180 mm; the angle between the two arms being 135°.

The motion of the valve may be assumed S.H.M., without dwell in fully open position.

Choose your own materials and suitable values for the stresses.

Draw fully dimensioned sketches of the valve gear.

Solution. Given: $d_r = 80$ mm; $h = 25$ mm; or $r = 25 / 2 = 12.5$ mm; $m = 0.0125$ m; $m = 0.4$ kg; $\alpha = 110°$; $N = 1500$ r.p.m.; $p_c = 0.4$ N/mm²; $p_s = 0.02$ N/mm²; $l = 180$ mm; $\theta = 135°$

A rocker arm for operating the exhaust valve is shown in Fig. 32.25.

First of all, let us find the various forces acting on the rocker arm of the exhaust valve.

We know that gas load on the valve,

$$P_1 = \frac{\pi}{4} (d_r)^2 p_c = \frac{\pi}{4} (80)^2 0.4 = 2011 \text{ N}$$

Weight of associated parts with the valve,

$$w = m \cdot g = 0.4 \times 9.8 = 3.92 \text{ N}$$

\[ P = P_1 + w = 2011 + 3.92 = 2014.92 \text{ N} \quad \ldots(i) \]

Initial spring force considering weight of the valve,

$$F_s = \frac{\pi}{4} (d_r)^2 p_s - w = \frac{\pi}{4} (80)^2 0.02 - 3.92 = 96.6 \text{ N} \quad \ldots(ii)$$

The force due to valve acceleration ($F_{va}$) may be obtained as discussed below:
We know that speed of camshaft

\[ \frac{N}{2} = \frac{1500}{2} = 750 \text{ r.p.m.} \]

and angle turned by the camshaft per second

\[ \frac{750}{60} \times 360 = 4500 \text{ deg/s} \]

\[ \therefore \text{Time taken for the valve to open and close, } t = \frac{\text{Angle of action of cam}}{\text{Angle turned by camshaft}} = \frac{110}{4500} = 0.024 \text{ s} \]

We know that maximum acceleration of the valve

\[ a = \omega^2 \cdot r = \left(\frac{2\pi}{t}\right)^2 r = \left(\frac{2\pi}{0.024}\right)^2 0.0125 = 857 \text{ m/s}^2 \ldots (\therefore \omega = \frac{2\pi}{t}) \]

\[ \therefore \text{Force due to valve acceleration, considering the weight of the valve, } F_a = m \cdot a + w = 0.4 \times 857 + 3.92 = 346.72 \text{ N} \ldots (iii) \]

and maximum load on the rocker arm for exhaust valve,

\[ F_e = P + F_s + F_a = 2014.92 + 96.6 + 346.72 = 2458.24 \text{ say 2460 N} \]

Since the length of the two arms of the rocker are equal, therefore, the load at the two ends of the arm are equal, \textit{i.e.} \( F_e = F_c = 2460 \text{ N} \).
We know that reaction at the fulcrum pin $F_r$,
\[ R_r = \sqrt{(F_c)^2 + (F_e)^2} = 2F_c \times F_e \times \cos \theta \]
\[ = \sqrt{(2460)^2 + (2460)^2} - 2 \times 2460 \times 2460 \times \cos 135^\circ = 4545 \text{ N} \]

Let us now design the various parts of the rocker arm.

1. **Design of fulcrum pin**

Let $d_1 = \text{Diameter of the fulcrum pin}$, and $l_1 = \text{Length of the fulcrum pin} = 1.25d_1 \ldots (\text{Assume})$

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin ($R_r$),
\[ 4545 = d_1 \times l_1 \times p_b = d_1 \times 1.25 \times 5 = 6.25 (d_1)^2 \]
...(For ordinary lubrication, $p_b$ is taken as 5 N/mm²)

\[ (d_1)^2 = \frac{4545}{6.25} = 727 \text{ or } d_1 = 26.97 \text{ say } 30 \text{ mm Ans.} \]

and \[ l_1 = 1.25d_1 = 1.25 \times 30 = 37.5 \text{ mm Ans.} \]

Now let us check the average shear stress induced in the pin. Since the pin is in double shear, therefore, load on the fulcrum pin ($R_{pl}$),
\[ 4545 = 2 \times \frac{\pi}{4} (d_1)^2 \tau = 2 \times \frac{\pi}{4} (30)^2 \tau = 1414 \tau \]

\[ \tau = \frac{4545}{1414} = 3.2 \text{ N/mm}^2 \text{ or MPa} \]

This induced shear stress is quite safe.

Now external diameter of the boss,
\[ D_1 = 2d_1 = 2 \times 30 = 60 \text{ mm} \]

Assuming a phosphor bronze bush of 3 mm thick, the internal diameter of the hole in the lever,
\[ d_{bh} = d_1 + 2 \times 3 = 30 + 6 = 36 \text{ mm} \]

Let us now check the induced bending stress for the section of the boss at the fulcrum which is shown in Fig. 32.26.
Bending moment at this section,
\[ M = F_c \times l = 2460 \times 180 = 443 \times 10^3 \text{ N-mm} \]
Section modulus,
\[ Z = \frac{1}{12} \times 37.5 \times [(60)^3 - (36)^3] \times 60/2 = 17640 \text{ mm}^3 \]
∴ Induced bending stress,
\[ \sigma_b = \frac{M}{Z} = \frac{443 \times 10^3}{17640} = 25.1 \text{ N/mm}^2 \text{ or MPa} \]
The induced bending stress is quite safe.

2. Design for forked end
Let 
\[ d_2 = \text{Diameter of the roller pin,} \]
and
\[ l_2 = \text{Length of the roller pin} \]
\[ = 1.25d_1 \] (Assume)

Considering bearing of the roller pin. We know that load on the roller pin \((F_c)\),
\[ 2460 = d_2 \times l_2 \times p_b = d_2 \times 1.25 \times 7 = 8.75 (d_2)^2 \]
∴ \( (d_2)^2 \) = \( \frac{2460}{8.75} = 281 \) or \( d_2 = 16.76 \) say 18 mm \text{ Ans.} 

\[ l_2 = 1.25 d_2 = 1.25 \times 18 = 22.5 \] say 24 mm \text{ Ans.} 

Let us now check the roller pin for induced shearing stress. Since the pin is in double shear, therefore, load on the roller pin \((F_c)\),
\[ 2460 = 2 \times \frac{\pi}{4} (d_2)^2 \tau = 2 \times \frac{\pi}{4} (18)^2 \tau = 509 \tau \]
∴ \( \tau = \frac{2460}{509} = 4.83 \) N/mm² or MPa

This induced shear stress is quite safe.

The roller pin is fixed in the eye and thickness of each eye is taken as one-half the length of the roller pin.
∴ Thickness of each eye,
\[ t_2 = \frac{l_2}{2} = \frac{24}{2} = 12 \text{ mm} \]

Let us now check the induced bending stress in the roller pin. The pin is neither simply supported in fork nor rigidly fixed at the end. Therefore, the common practice is to assume the load distribution as shown in Fig. 32.27.

The maximum bending moment will occur at \(Y-Y\).
Neglecting the effect of clearance, we have
Maximum bending moment at \(Y-Y\),
\[ M = \frac{F_c}{2} \left( \frac{l_2}{2} + \frac{t_2}{3} \right) - \frac{F_c}{4} \times \frac{l_2}{4} \]
\[ = \frac{F_c}{2} \left( \frac{l_2}{2} + \frac{t_2}{6} \right) - \frac{F_c}{4} \times \frac{l_2}{4} \]
\[ = \frac{5}{24} \times F_c \times l_2 = \frac{5}{24} \times 2460 \times 24 \]
\[ = 12300 \text{ N-mm} \]
and section modulus of the pin,

\[ Z = \frac{\pi}{32} (d_2)^3 = \frac{\pi}{32} (18)^3 = 573 \text{ mm}^3 \]

\[ \therefore \text{ Bending stress induced in the pin} \]

\[ = \frac{M}{Z} = \frac{12300}{573} = 21.5 \text{ N/mm}^2 \text{ or MPa} \]

This bending stress induced in the pin is within permissible limits.

Since the radial thickness of eye \( (t_3) \) is taken as \( d_2/2 \), therefore, overall diameter of the eye,

\[ D_2 = 2 \times d_2 = 2 \times 18 = 36 \text{ mm} \]

The outer diameter of the roller is taken slightly larger (atleast 3 mm more) than the outer diameter of the eye.

In the present case, 42 mm outer diameter of the roller will be sufficient.

Providing a clearance of 1.5 mm between the roller and the fork on either side of the roller, we have

\[ l_3 = l_2 + 2 \times \frac{t_2}{2} + 2 \times 1.5 \]

\[ = 24 + 2 \times \frac{12}{2} + 3 = 39 \text{ mm} \]

3. Design for rocker arm cross-section

The cross-section of the rocker arm is obtained by considering the bending of the sections just near the boss of fulcrum on both sides, such as section \( A - A \) and \( B - B \).

We know that maximum bending moment at \( A - A \) and \( B - B \).

\[ M = 2460 \left( 180 - \frac{60}{2} \right) = 369 \times 10^3 \text{ N-mm} \]

The rocker arm is of \( I \)-section. Let us assume the proportions as shown in Fig. 32.28. We know that section modulus,

\[ Z = \frac{1}{12} \left[ \frac{2.5t}{6t/2} (6t)^3 - 1.5t(4t)^3 \right] = \frac{37t^4}{3t} = 12.33 t^3 \]

\[ \therefore \text{ Bending stress} (\sigma_b), \]

\[ 70 = \frac{M}{Z} = \frac{369 \times 10^3}{12.33t^3} = \frac{29.93 \times 10^3}{t^3} \]

\[ t^3 = 29.93 \times 10^3 / 70 = 427.6 \text{ or } t = 7.5 \text{ say 8 mm} \]

\[ \therefore \text{ Width of flange} = 2.5 \times 2.5 \times 8 = 20 \text{ mm Ans.} \]

Depth of web = 4 \( t = 4 \times 8 = 32 \text{ mm Ans.} \]

and depth of the section = 6 \( t = 6 \times 8 = 48 \text{ mm Ans.} \)

Normally thickness of the flange and web is constant throughout, whereas the width and depth is tapered.

4. Design for tappet screw

The adjustable tappet screw carries a compressive load of \( F_e = 2460 \text{ N} \). Assuming the screw is made of mild steel for which the compressive stress \( (\sigma_c) \) may be taken as 50 MPa.
Let \( d_c \) = Core diameter of the tappet screw.

We know that the load on the tappet screw \( (F_c) \),
\[
2460 = \frac{\pi}{4} (d_c)^2 \sigma_c = \frac{\pi}{4} (d_c)^2 \times 50 = 39.3 \ (d_c)^2
\]

\[
\therefore \quad (d_c)^2 = \frac{2460}{39.3} = 62.6 \quad \text{or} \quad d_c = 7.9 \text{ say } 8 \text{ mm}
\]

and outer or nominal diameter of the screw,
\[
d = \frac{d_c}{0.84} = \frac{8}{0.84} = 9.52 \text{ say } 10 \text{ mm \ Ans.}
\]

We shall use 10 mm stud and it is provided with a lock nut. The diameter of the circular end of the arm \( (D_3) \) and its depth \( (t_4) \) is taken as twice the diameter of stud.

\[
\therefore \quad D_3 = 2 \times 10 = 20 \text{ mm \ Ans.}
\]

\[
\text{and } \quad t_4 = 2 \times 10 = 20 \text{ mm \ Ans.}
\]

5. Design for valve spring

First of all, let us find the total load on the valve spring.

We know that initial load on the spring,
\[
W_1 = \text{Initial spring force } (F_s) = 96.6 \text{ N} \quad \text{ ...(Already calculated)}
\]

and load at full lift,
\[
W_2 = \text{Full valve lift } \times \text{ Stiffness of spring } (s)
\]
\[
= 25 \times 10 = 250 \text{ N} \quad \text{ ...(Assuming } s = 10 \text{ N/mm)}
\]

\[
\therefore \quad \text{Total load on the spring,}
\]
\[
W = W_1 + W_2 = 96.6 + 250 = 346.6 \text{ N}
\]

Now let us find the various dimensions for the valve spring, as discussed below:

(a) Mean diameter of spring coil

Let \( D \) = Mean diameter of the spring coil, and
\( d \) = Diameter of the spring wire.

We know that Wahl’s stress factor,
\[
K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184
\]

\[
\text{...(Assuming } C = D/d = 8)
\]

and maximum shear stress \( (\tau) \),
\[
420 = K \times \frac{8WC}{\pi d^2} = 1.184 \times \frac{8 \times 346.6 \times 8}{\pi d^2} = \frac{8360}{d^2}
\]

\[
\text{...(Assuming } \tau = 420 \text{ MPa or N/mm}^2)
\]

\[
\therefore \quad d^2 = \frac{8360}{420} = 19.9 \quad \text{or} \quad d = 4.46 \text{ mm}
\]

The standard size of the wire is SWG 7 having diameter \( (d) \) = 4.47 mm. \textbf{Ans.} (See Table 22.2).

\[
\therefore \quad \text{Mean diameter of the spring coil,}
\]
\[
D = C \cdot d = 8 \times 4.47 = 35.76 \text{ mm \ Ans.}
\]

and outer diameter of the spring coil,
\[
D_o = D + d = 35.76 + 4.47 = 40.23 \text{ mm \ Ans.}
\]

(b) Number of turns of the coil

Let \( n \) = Number of active turns of the coil.

We know that maximum compression of the spring,
\[
\delta = \frac{8W \cdot C^3 \cdot n}{G \cdot d} \quad \text{ or } \quad \frac{\delta}{W} = \frac{8C^3 \cdot n}{G \cdot d}
\]
Since the stiffness of the springs, \( s = W / \delta = 10 \text{ N/mm} \), therefore, \( \delta / W = 1/10 \). Taking \( G = 84 \times 10^3 \text{ MPa or N/mm}^2 \), we have
\[
\frac{1}{10} = \frac{8 \times 8^3 \times n}{84 \times 10^3 \times 4.47} = \frac{10.9}{n} \times 10^3
\]
\[ \therefore \quad n = 10^3 / 10.9 \times 10 = 9.17 \text{ say 10} \]
For squared and ground ends, the total number of the turns, \( n' = n + 2 = 10 + 2 = 12 \text{ Ans.} \)

(c) **Free length of the spring**
Since the compression produced under \( W_2 = 250 \text{ N} \) is 25 mm (i.e. equal to full valve lift), therefore, maximum compression produced (\( \delta_{\text{max}} \)) under the maximum load of \( W = 346.6 \text{ N} \) is
\[
\delta_{\text{max}} = \frac{25}{250} \times 346.6 = 34.66 \text{ mm}
\]
We know that free length of the spring,
\[
L_f = n' \cdot d + \delta_{\text{max}} + 0.15 \delta_{\text{max}}
\]
\[ = 12 \times 4.47 + 34.66 + 0.15 \times 34.66 = 93.5 \text{ mm Ans.} \]

(d) **Pitch of the coil**
We know that pitch of the coil
\[
= \frac{\text{Free length}}{n' - 1} = \frac{93.5}{12 - 1} = 8.5 \text{ mm Ans.} \]

**Example 32.8.** Design the various components of the valve gear mechanism for a horizontal diesel engine for the following data:

- **Bore = 140 mm**
- **Stroke = 270 mm**
- **Power = 8.25 kW**
- **Speed = 475 r.p.m.**
- **Maximum gas pressure = 3.5 N/mm²**
The valve opens $33^\circ$ before outer dead centre and closes $1^\circ$ after inner dead centre. It opens and closes with constant acceleration and decleration for each half of the lift. The length of the rocker arm on either side of the fulcrum is 150 mm and the included angle is $160^\circ$. The weight of the valve is 3 N.

**Solution.** Given: $D = 140$ mm = 0.14 m; $L = 270$ mm = 0.27 m; Power = 8.25 kW = 8250 W; $N = 475$ r.p.m; $p = 3.5$ N/mm$^2$; $l = 150$ mm = 0.15 m; $\theta = 160^\circ$; $w = 3$ N

First of all, let us find out dimensions of the valve as discussed below:

**Size of the valve port**

Let $d_p$ = Diameter of the valve port, and 

$\quad a_p = \text{Area of the valve port} = \frac{\pi}{4} (d_p)^2$

We know that area of the piston,

$\quad a = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.14)^2 = 0.0154$ m$^2$

and mean velocity of the piston,

$\quad v = \frac{2LN}{60} = \frac{2 \times 0.27 \times 475}{60} = 4.275$ m/s

From Table 32.3, let us take the mean velocity of the gas through the port ($v_p$) as 40 m/s.

We know that $a_p \cdot v_p = a \cdot v$

$\quad \frac{\pi}{4} (d_p)^2 40 = 0.0154 \times 4.275$ or $31.42 (d_p)^2 = 0.0658$

$\therefore (d_p)^2 = \frac{0.0658}{31.42} = 2.09 \times 10^{-3}$ or $d_p = 0.045$ m = 45 mm Ans.

**Maximum lift of the valve**

We know that maximum lift of the valve,

$\quad h = \frac{d_p}{4 \cos \alpha} = \frac{45}{4 \cos 45^\circ} = 15.9$ say 16 mm Ans.

...(Taking $\alpha = 45^\circ$)

**Thickness of the valve head**

We know that thickness of valve head,

$\quad t = k \cdot d_p \sqrt{\frac{p}{\sigma_b}} = 0.42 \times 45 \sqrt{\frac{3.5}{56}} = 4.72$ mm Ans.

...(Taking $k = 0.42$ and $\sigma_b = 56$ MPa)

**Valve stem diameter**

We know that valve stem diameter,

$\quad d_s = \frac{d_p}{8} + 6.35$ mm = $\frac{45}{8} + 6.35 = 11.97$ say 12 mm Ans.

**Valve head diameter**

The projected width of the valve seat, for a seat angle of $45^\circ$, may be empirically taken as $0.05 d_p$ to $0.07 d_p$. Let us take width of the valve seat as $0.06 d_p$, i.e. $0.06 \times 45 = 2.7$ mm.

$\therefore$ Valve head diameter, $d_v = d_p + 2 \times 2.7 = 45 + 5.4 = 50.4$ say 51 mm Ans.

Now let us calculate the various forces acting on the rocker arm of exhaust valve.
We know that gas load on the valve,

\[ P_1 = \frac{\pi}{4} (d_c^2) p_c = \frac{\pi}{4} (51)^2 0.4 = 817 \text{ N} \]  
...(Taking \( p_c = 0.4 \text{ N/mm}^2 \))

Total load on the valve, considering the weight of the valve,

\[ P = P_1 + w = 817 + 3 = 820 \text{ N} \]

Initial spring force, considering the weight of the valve,

\[ F_s = \frac{\pi}{4} (d_s^2) p_s - w = \frac{\pi}{4} (51)^2 0.025 - 3 = 48 \text{ N} \]  
...(Taking \( p_s = 0.025 \text{ N/mm}^2 \))

The force due to acceleration (\( F_a \)) may be obtained as discussed below:

We know that total angle of crank for which the valve remains open

\[ = 33 + 180 + 1 = 214^\circ \]

Since the engine is a four stroke engine, therefore the camshaft angle for which the valve remains open

\[ = 214 / 2 = 107^\circ \]

Now, when the camshaft turns through 107 / 2 = 53.5°, the valve lifts by a distance of 16 mm. It may be noted that the half of this period is occupied by constant acceleration and half by constant decleration. The same process occurs when the valve closes. Therefore, the period for constant acceleration is equal to camshaft rotation of 53.5 / 2 = 26.75 ° and during this time, the valve lifts through a distance of 8 mm.

We know that speed of camshaft

\[ = \frac{N}{2} = \frac{475}{2} = 237.5 \text{ r.p.m.} \]

\[ \therefore \text{Angle turned by the camshaft per second} \]

\[ = \frac{237.5}{60} \times 360 = 1425 \text{ deg / s} \]

and time taken by the camshaft for constant acceleration,

\[ t = \frac{26.75}{1425} = 0.0188 \text{ s} \]

Let

\[ a = \text{Acceleration of the valve.} \]

We know that

\[ s = u . t + \frac{1}{2} a . t^2 \]  
...(Equation of motion)

\[ 8 = 0 \times t + \frac{1}{2} a (0.0188)^2 = 1.767 \times 10^{-4} a \]  
...(\( u = 0 \))

\[ \therefore \quad a = \frac{8}{1.767 \times 10^{-4}} = 45,274 \text{ mm / s}^2 = 45.274 \text{ m / s}^2 \]

and force due to valve acceleration, considering the weight of the valve,

\[ F_a = m \cdot a + w = \frac{3}{9.81} \times 45.274 + 3 = 16.84 \text{ N} \]  
...(\( m = w/g \))

We know that the maximum load on the rocker arm for exhaust valve,

\[ F_e = P + F_s + F_a = 820 + 48 + 16.84 = 884.84 \text{ say 885 N} \]

Since the length of the two arms of the rocker are equal, therefore, load at the two ends of the arm are equal, \( i.e. F_e = F_c = 885 \text{ N} \).
We know that reaction at the fulcrum pin $F_r$

$$R_r = \sqrt{F_r^2 + F_c^2 - 2F_r \times F_c \times \cos \theta}$$

$$= \sqrt{(885)^2 + (885)^2 - 2 \times 885 \times 885 \times \cos 160^\circ} = 1743 \text{ N}$$

The rocker arm is shown in Fig. 32.29. We shall now design the various parts of rocker arm as discussed below:

1. **Design of fulcrum pin**

Let $d_1 =$ Diameter of the fulcrum pin, and $l_1 =$ Length of the fulcrum pin $= 1.25 \times d_1$ ... (Assume)

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin $(R_r)$,$

$$1743 = d_1 \times l_1 \times p_b = d_1 \times 1.25 \times d_1 \times 5 = 6.25 \times (d_1)^2$$

...(For ordinary lubrication, $p_b$ is taken as 5 N/mm$^2$)

$\therefore$ \hspace{1cm} $(d_1)^2 = 1743 / 6.25 = 279$ or $d_1 = 16.7$ say 17 mm

and

$$l_1 = 1.25 \times d_1 = 1.25 \times 17 = 21.25$$ say 22 mm

Now let us check the average shear stress induced in the pin. Since the pin is in double shear, therefore, load on the fulcrum pin $(R_r)$,

$$1743 = 2 \times \frac{\pi}{4} (d_1)^2 \tau = 2 \times \frac{\pi}{4} (17)^2 \times 17 = 454 \times \tau$$

$\therefore$ \hspace{1cm} $\tau = 1743 / 454 = 3.84 \text{ N/mm}^2$ or MPa

This induced shear stress is quite safe.

Now external diameter of the boss,

$$D_1 = 2d_1 = 2 \times 17 = 34 \text{ mm}$$

Assuming a phosphor bronze bush of 3 mm thick, the internal diameter of the hole in the lever,

$$d_h = d_1 + 2 \times 3 = 17 + 6 = 23 \text{ mm}$$
Now, let us check the induced bending stress for the section of the boss at the fulcrum which is shown in Fig. 32.30.

Bending moment at this section,
\[ M = F_{c} \times l = 885 \times 150 \text{ N-mm} \]
\[ = 132,750 \text{ N-mm} \]
Section modulus,
\[ Z = \frac{1}{12} \times 22 \left[ \frac{(34)^3}{34/2} - \frac{(23)^3}{23/2} \right] = 2927 \text{ mm}^3 \]
∴ Induced bending stress,
\[ \sigma_b = \frac{M}{Z} = \frac{132,750}{2927} = 45.3 \text{ N/mm}^2 \text{ or MPa} \]
The induced bending stress is quite safe.

2. Design for forked end

Let \( d_2 \) = Diameter of the roller pin, and
\[ l_2 = \text{Length of the roller pin} = 1.25 \times d_2 \] ...(Assume)

Considering bearing of the roller pin. We know that load on the roller pin \((F_c)\),
\[ 885 = d_2 \times l_2 \times p_b = d_2 \times 1.25 \times 7 = 8.75 \times (d_2)^2 \]
...(Taking \( p_b = 7 \text{ N/mm}^2 \))
∴ \( (d_2)^2 = \frac{885}{8.75} = 101.14 \) or \( d_2 = 10.06 \text{ say 11 mm} \text{ Ans.} \)
and \[ l_2 = 1.25 \times d_2 = 1.25 \times 11 = 13.75 \text{ say 14 mm} \text{ Ans.} \]
Let us now check the roller pin for induced shearing stress. Since the pin is in double shear, therefore, load on the roller pin \( F_c \),

\[
885 = 2 \times \frac{\pi}{4} \left( d_2 \right)^2 \tau = 2 \times \frac{\pi}{4} (11)^2 \tau = 190 \ \tau
\]

\[
\therefore \quad \tau = \frac{885}{190} = 4.66 \text{ N/mm}^2 \text{ or MPa}
\]

This induced shear stress is quite safe.

The roller pin is fixed in the eye and thickness of each eye is taken as one-half the length of the roller pin.

\[
\therefore \text{Thickness of each eye,} \quad t_2 = \frac{l_2}{2} = \frac{14}{2} = 7 \text{ mm}
\]

Let us now check the induced bending stress in the roller pin. The pin is neither simply supported in fork nor rigidly fixed at the end. Therefore, the common practice is to assume the load distribution as shown in Fig. 32.31.

The maximum bending moment will occur at \( Y-Y \).

Neglecting the effect of clearance, we have

Maximum bending moment at \( Y-Y \),

\[
M = \frac{F_c}{2} \left( \frac{l_2}{2} + \frac{t_2}{3} \right) - \frac{F_c}{2} \times \frac{l_2}{4}
\]

\[
= \frac{5}{24} \times F_c \times l_2
\]

\[
= \frac{5}{24} \times 885 \times 14 = 2581 \text{ N-mm}
\]

and section modulus of the pin,

\[
Z = \frac{\pi}{32} (d_2)^3 = \frac{\pi}{32} (11)^3 = 131 \text{ mm}^3
\]

\[
\therefore \text{Bending stress induced in the pin} \quad \frac{M}{Z} = \frac{2581}{131} = 19.7 \text{ N/mm}^2 \text{ or MPa}
\]

This bending stress induced in the pin is within permissible limits.

Since the radial thickness of eye \( t_3 \) is taken as \( d_2 / 2 \), therefore, overall diameter of the eye,

\[
D_2 = 2 \times \frac{d_2}{2} = 2 \times 11 = 22 \text{ mm}
\]

The outer diameter of the roller is taken slightly larger (at least 3 mm more) than the outer diameter of the eye. In the present case, 28 mm outer diameter of the roller will be sufficient.

Providing a clearance of 1.5 mm between the roller and the fork on either side of the roller, we have

\[
l_3 = l_2 + 2 \times \frac{t_2}{2} + 2 \times 1.5 = 14 + 2 \times \frac{7}{2} + 3 = 24 \text{ mm}
\]

3. **Design for rocker arm cross-section**

Since the engine is a slow speed engine, therefore, a rectangular section may be selected for the rocker arm. The cross-section of the rocker arm is obtained by considering the bending of the sections just near the boss of fulcrum on both sides, such as section \( A-A \) and \( B-B \).
Let \( t_1 \) = Thickness of the rocker arm which is uniform throughout.
\( B \) = Width or depth of the rocker arm which varies from boss diameter of fulcrum to outside diameter of the eye (for the forked end side) and from boss diameter of fulcrum to thickness \( t_3 \) (for the tappet or stud end side).

Now bending moment on section \( A – A \) and \( B – B \),
\[
M = 885 \left(150 - \frac{34}{2}\right) = 117,705 \text{ N-mm}
\]
and section modulus at \( A – A \) and \( B – B \),
\[
Z = \frac{1}{6} \times t_1 \cdot B^2 = \frac{1}{6} \times t_1 \left(D_3\right)^2 = \frac{1}{6} \times t_1 \left(34\right)^2 = 193 \ t_1
\]
\[
\text{(At sections } A-A \text{ and } B-B, \ B = D)\]

We know that bending stress \( (\sigma_b) \),
\[
70 = \frac{M}{Z} = \frac{117,705}{193 \ t_1}
\]
\[
\therefore \ t_1 = \frac{117,705}{193 \times 70} = 8.7 \text{ say } 10 \text{ mm Ans.}
\]

4. Design for tappet screw
The adjustable tappet screw carries a compressive load of \( F_e = 885 \) N. Assuming the screw to be made of mild steel for which the compressive stress \( (\sigma_c) \) may be taken as 50 MPa.

Let \( d_c \) = Core diameter of the tappet screw.

We know that load on the tappet screw \( (F_e) \),
\[
885 = \frac{\pi}{4} \left(d_c^2\right) \sigma_c = \frac{\pi}{4} \left(d_c^2\right) 50 = 39.3 \left(d_c\right)^2
\]
\[
\therefore \ (d_c)^2 = \frac{885}{39.3} = 22.5 \text{ or } d_c = 4.74 \text{ say } 5 \text{ mm Ans.}
\]
and outer or nominal diameter of the screw,
\[
d = \frac{d_c}{0.84} = \frac{5}{0.84} = 6.25 \text{ say } 6.5 \text{ mm Ans.}
\]

We shall use 6.5 mm stud and it is provided with a lock nut. The diameter of the circular end of the arm \( (D_3) \) and its depth \( (t_4) \) is taken as twice the diameter of stud.
\[
\therefore \ D_3 = 2 \times 6.5 = 13 \text{ mm Ans.}
\]
and \( t_4 = 2 \times 6.5 = 13 \text{ mm Ans.}
\]

5. Design for valve spring
First of all, let us find the total load on the valve spring.

We know that initial load on the spring,
\[
W_1 = \text{Initial spring force } (F_e) = 48 \text{ N} \quad \text{(Already calculated)}
\]
and load at full lift,
\[
W_2 = \text{Full valve lift } \times \text{ Stiffness of spring } (s) = 16 \times 8 = 128 \text{ N} \quad \text{(Taking } s = 8 \text{ N/mm)}
\]
\[
\therefore \ \text{Total load on the spring,} \ W = W_1 + W_2 = 48 + 128 = 176 \text{ N}
\]

Now let us find the various dimensions for the valve spring as discussed below:

(a) Mean diameter of the spring coil
Let \( D \) = Mean diameter of the spring coil, and \( d \) = Diameter of the spring wire.

\[
W = W_1 + W_2 = 48 + 128 = 176 \text{ N}
\]
We know that Wahl’s stress factor,

\[ K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525 \]

...(Assuming \( C = D/d = 6 \))

and maximum shear stress (\( \tau \)),

\[ 420 = K \times \frac{8WC}{\pi d^2} = 1.2525 \times \frac{8 \times 176 \times 6}{\pi d^2} = \frac{3368}{d^2} \]

\[ \therefore \quad d^2 = \frac{3368}{420} = 8.02 \quad \text{or} \quad d = 2.83 \text{ mm} \]

The standard size of the wire is SWG 11 having a diameter \( (d) = 2.946 \text{ mm} \) Ans. (see Table 22.2)

\[ \therefore \quad \text{Mean diameter of spring coil,} \quad D = C \cdot d = 6 \times 2.946 = 17.676 \text{ mm} \quad \text{Ans.} \]

and outer diameter of the spring coil,

\[ D_o = D + d = 17.676 + 2.946 = 20.622 \text{ mm} \quad \text{Ans.} \]

(b) **Number of turns of the coil**

Let \( n = \) Number of turns of the coil,

We know that maximum compression of the spring.

\[ \delta = \frac{8W \cdot C^3 \cdot n}{G \cdot d} \quad \text{or} \quad \frac{\delta}{W} = \frac{8C^3 \cdot n}{G \cdot d} \]
Since the stiffness of the spring, \( s = \frac{W}{\delta} = 8 \text{ N/mm} \), therefore \( \frac{\delta}{W} = \frac{1}{8} \). Taking \( G = 84 \times 10^3 \text{ MPa or N/mm}^2 \), we have

\[
\frac{1}{8} = \frac{8 \times 6^3 \times n}{84 \times 10^3 \times 2.946} = \frac{6.98 n}{10^3}
\]

\[
\therefore \quad n = \frac{10^3}{8} \times 6.98 = 17.9 \text{ say 18}
\]

For squared and ground ends, the total number of turns,

\[
n' = n + 2 = 18 + 2 = 20 \text{ Ans.}
\]

(c) **Free length of the spring**

Since the compression produced under \( W = 128 \text{ N} \) is 16 mm, therefore, maximum compression produced under the maximum load of \( W = 176 \text{ N} \) is

\[
\delta_{\text{max}} = \frac{16}{128} \times 176 = 22 \text{ mm}
\]

We know that free length of the spring,

\[
L'_{F} = n'. d + \delta_{\text{max}} + 0.15 \delta_{\text{max}}
\]

\[
= 20 \times 2.946 + 22 + 0.15 \times 22 = 84.22 \text{ say 85 mm Ans.}
\]

(d) **Pitch of the coil**

We know that pitch of the coil

\[
\text{Pitch} = \frac{\text{Free length}}{n' - 1} = \frac{85}{20 - 1} = 4.47 \text{ mm Ans.}
\]

**Design of cam**

The cam is forged as one piece with the camshaft. It is designed as discussed below:

The diameter of camshaft (\( D' \)) is taken empirically as

\[
D' = 0.16 \times \text{Cylinder bore} + 12.7 \text{ mm}
\]

\[
= 0.16 \times 140 + 12.7 = 35.1 \text{ say 36 mm}
\]

The base circle diameter is about 3 mm greater than the camshaft diameter.

\[
\therefore \quad \text{Base circle diameter} = 36 + 3 = 39 \text{ say 40 mm}
\]

The width of cam is taken equal to the width of roller, \( i.e. \) 14 mm.

The width of cam (\( w' \)) is also taken empirically as

\[
w' = 0.09 \times \text{Cylinder bore} + 6 \text{ mm} = 0.09 \times 140 + 6 = 18.6 \text{ mm}
\]

Let us take the width of cam as 18 mm.

Now the cam is drawn according to the procedure given below:

First of all, the displacement diagram, as shown in Fig. 32.32, is drawn as discussed in the following steps:

1. Draw a horizontal line \( ANM \) such that \( AN \) represents the angular displacement when valve opens (\( i.e. \) 53.5º) to some suitable scale. The line \( NM \) represents the angular displacement of the cam when valve closes (\( i.e. \) 53.5º).
2. Divide \( AN \) and \( NM \) into any number of equal even parts (say six).
3. Draw vertical lines through points 0, 1, 2, 3 etc. equal to the lift of valve, \( i.e. \) 16 mm.
4. Divide the vertical lines 3 – \( f \) and 3′ – \( f' \) into six equal parts as shown by points \( a, b, c \ldots \) and \( a', b', c' \ldots \) in Fig. 32.32.
5. Since the valve moves with equal uniform acceleration and decleration for each half of the lift, therefore, valve displacement diagram for opening and closing of valve consists of double parabola.

* For complete details, refer Authors’ popular book on ‘Theory of Machines’. 
6. Join $Aa$, $Ab$, $Ac$ intersecting the vertical lines through 1, 2, 3 at $B$, $C$, $D$ respectively.

7. Join the points $B$, $C$, $D$ with a smooth curve. This is the required parabola for the half of valve opening. Similarly other curves may be drawn as shown in Fig. 32.32.

8. The curve $A$, $B$, $C$, ..., $G$, $K$, $L$, $M$ is the required displacement diagram.

Now the profile of the cam, as shown in Fig. 32.32, is drawn as discussed in the following steps:

1. Draw a base circle with centre $O$ and diameter equal 40 mm (radius = 40/2 = 20 mm)

2. Draw a prime circle with centre $O$ and radius, $OA$ = Min. radius of cam + $\frac{1}{2}$

   Diameter of roller = $20 + \frac{1}{2} \times 28 = 20 + 14 = 34$ mm

3. Draw angle $AOG = 53.5^\circ$ to represent opening of valve and angle $GOM = 53.5^\circ$ to represent closing of valve.

4. Divide the angular displacement of the cam during opening and closing of the valve (i.e. angle $AOG$ and $GOM$) into same number of equal even parts as in displacement diagram.

5. Join the points 1, 2, 3, etc. with the centre $O$ and produce the lines beyond prime circle as shown in Fig. 32.33.

6. Set off points $1B$, $2C$, $3D$, etc. equal to the displacements from displacement diagram.

7. Join the points $A$, $B$, $C$,...,$L$, $M$, $A$. The curve drawn through these points is known as pitch curve.

8. From the points $A$, $B$, $C$, ...$K$, $L$, draw circles of radius equal to the radius of the roller.
9. Join the bottoms of the circle with a smooth curve as shown in Fig. 32.33. This is the required profile of cam.

**EXERCISES**

1. A four stroke internal combustion engine has the following specifications:
   Brake power = 7.5 kW; Speed = 1000 r.p.m.; Indicated mean effective pressure = 0.35 N/mm²; Maximum gas pressure = 3.5 N/mm²; Mechanical efficiency = 80%.
   Determine: 1. The dimensions of the cylinder, if the length of stroke is 1.4 times the bore of the cylinder; 2. Wall thickness of the cylinder, if the hoop stress is 35 MPa; 3. Thickness of the cylinder head and the size of studs when the permissible stresses for the cylinder head and stud materials are 45 MPa and 65 MPa respectively.

2. Design a cast iron trunk type piston for a single acting four stroke engine developing 75 kW per cylinder when running at 600 r.p.m. The other available data is as follows:
   Maximum gas pressure = 4.8 N/mm²; Indicated mean effective pressure = 0.65 N/mm²; Mechanical efficiency = 95%; Radius of crank = 110 mm; Fuel consumption = 0.3 kg/BP/hr; Calorific value of fuel (higher) = 44 × 10³ kJ/kg; Difference of temperatures at the centre and edges of the piston head = 200°C; Allowable stress for the material of the piston = 33.5 MPa; Allowable stress for the material of the piston rings and gudgeon pin = 80 MPa; Allowable bearing pressure on the piston barrel = 0.4 N/mm² and allowable bearing pressure on the gudgeon pin = 17 N/mm².

3. Design a piston for a four stroke diesel engine consuming 0.3 kg of fuel per kW of power per hour and produces a brake mean effective pressure of the 0.7 N/mm². The maximum gas pressure inside the cylinder is 5 N/mm² at a speed of 3500 r.p.m. The cylinder diameter is required to be 300 mm with stroke 1.5 times the diameter. The piston may have 4 compression rings and an oil ring. The following data can be used for design:
Higher calorific value of fuel = $46 \times 10^3$ kJ/kg; Temperature at the piston centre = 700 K; Temperature at the piston edge = 475 K; Heat conductivity factor = 46.6 W/mK; Heat conducted through top = 5% of heat produced; Permissible tensile strength for the material of piston = 27 N/mm²; Pressure between rings and piston = 0.04 N/mm²; Permissible tensile stress in rings = 80 N/mm²; Permissible pressure on piston barrel = 0.4 N/mm²; Permissible pressure on piston pin = 15 N/mm²; Permissible stress in piston pin = 85 N/mm².

Any other data required for the design may be assumed.

4. Determine the dimensions of an I-section connecting rod for a petrol engine from the following data:
   - Diameter of the piston = 110 mm;
   - Mass of the reciprocating parts = 2 kg;
   - Length of the connecting rod from centre to centre = 325 mm;
   - Stroke length = 150 mm;
   - R.P.M. = 1500 with possible overspeed of 2500;
   - Compression ratio = 4 : 1;
   - Maximum explosion pressure = 2.5 N/mm².

5. The following particulars refer to a four stroke cycle diesel engine:
   - Cylinder bore = 150 mm;
   - Stroke = 187.5 mm;
   - R.P.M. = 1200;
   - Maximum gas pressure = 5.6 N/mm²;
   - Mass of reciprocating parts = 1.75 kg.
   - The dimensions of an I-section connecting rod of forged steel with an elastic limit compressive stress of 350 MPa. The ratio of the length of connecting rod to the length of crank is 4 and the factor of safety may be taken as 5;
   - The wrist pin and crankpin dimensions on the basis of bearing pressures of 10 N/mm² and 6.5 N/mm² of the projected area respectively; and
   - The dimensions of the small and big ends of the connecting rods, including the size of the securing bolts of the crankpin end. Assume that the allowable stress in the bolts, is not to exceed 35 N/mm².

Draw dimensioned sketches of the connecting rod showing the provisions for lubrication.

6. A connecting rod is required to be designed for a high speed, four stroke I.C. engine. The following data are available.
   - Diameter of piston = 88 mm;
   - Mass of reciprocating parts = 1.6 kg;
   - Length of connecting rod (centre to centre) = 300 mm;
   - Stroke = 125 mm;
   - R.P.M. = 2200 (when developing 50 kW);
   - Possible overspeed = 3000 r.p.m.;
   - Compression ratio = 6.8 : 1 (approximately);
   - Probable maximum explosion pressure (assumed shortly after dead centre, say at about 3°) = 3.5 N/mm².

Draw fully dimensioned drawings of the connecting rod showing the provision for the lubrication.

7. Design a plain carbon steel centre crankshaft for a single acting four stroke, single cylinder engine for the following data:
   - Piston diameter = 250 mm;
   - Stroke = 400 mm;
   - Maximum combustion pressure = 2.5 N/mm²;
   - Weight of the flywheel = 16 kN;
   - Total belt pull = 3 N;
   - Length of connecting rod = 950 mm.

When the crank has turned through 30° from top dead centre, the pressure on the piston is 1 N/mm² and the torque on the crank is maximum.

Any other data required for the design may be assumed.

8. Design a side crankshaft for a 500 mm × 600 mm gas engine. The weight of the flywheel is 80 kN and the explosion pressure is 2.5 N/mm². The gas pressure at maximum torque is 0.9 N/mm² when the crank angle is 30°. The connecting rod is 4.5 times the crank radius.

Any other data required for the design may be assumed.

9. Design a rocker arm of I-section made of cast steel for operating an exhaust valve of a gas engine. The effective length of the rocker arm is 250 mm and the angle between the arm is 135°. The exhaust valve is 80 mm in diameter and the gas pressure when the valve begins to open is 0.4 N/mm². The greatest suction pressure is 0.03 N/mm² below atmospheric. The initial load may be assumed as 0.05 N/mm² of valve area and the valve inertia and friction losses as 120 N. The ultimate strength of cast steel is 750 MPa. The allowable bearing pressure is 8 N/mm² and the permissible stress in the material is 72 MPa.

10. Design the various components of a valve gear mechanism for a horizontal diesel engine having the following specifications:
Brake power = 10 kW; Bore = 140 mm; Stroke = 270 mm; Speed = 500 r.p.m. and maximum gas pressure = 3.5 N/mm².
The valve opens 30° before top dead centre and closes 2° after bottom dead centre. It opens and closes with constant acceleration and deceleration for each half of the lift. The length of the rocker arm on either side of the fulcrum is 150 mm and the included angle is 135°. The mass of the valve is 0.3 kg.

QUESTIONS

1. Explain the various types of cylinder liners.
2. Discuss the design of piston for an internal combustion engine.
3. State the function of the following for an internal combustion engine piston:
   (a) Ribs; (b) Piston rings; (c) Piston skirt; and (d) Piston pin
4. What is the function of a connecting rod of an internal combustion engine?
5. Explain the various stresses induced in the connecting rod.
6. Under what force, the big end bolts and caps are designed?
7. Explain the various types of crankshafts.
8. At what angle of the crank, the twisting moment is maximum in the crankshaft?
9. What are the methods and materials used in the manufacture of crankshafts?
10. Sketch a valve gear mechanism of an internal combustion engine and label its various parts.
11. Discuss the materials commonly used for making the valve of an I.C. engine.
12. Why the area of the inlet valve port is made larger than the area of exhaust valve port?
OBJECTIVE TYPE QUESTIONS

1. The cylinders are usually made of
   (a) cast iron or cast steel   (b) aluminium
   (c) stainless steel   (d) copper
2. The length of the cylinder is usually taken as
   (a) equal to the length of piston   (b) equal to the length of stroke
   (c) equal to the cylinder bore   (d) 1.5 times the length of stroke
3. The skirt of piston
   (a) is used to withstand the pressure of gas in the cylinder
   (b) acts as a bearing for the side thrust of the connecting rod
   (c) is used to seal the cylinder in order to prevent leakage of the gas past the piston
   (d) none of the above
4. The side thrust on the cylinder liner is usually taken as .......... of the maximum gas load on the piston.
   (a) 1/5   (b) 1/8
   (c) 1/10   (d) 1/5
5. The length of the piston usually varies between
   (a) $D \text{ and } 1.5 \times D$   (b) $1.5 \times D \text{ and } 2 \times D$
   (c) $2 \times D \text{ and } 2.5 \times D$   (d) $2.5 \times D \text{ and } 3 \times D$
   where $D =$ Diameter of the piston.
6. In designing a connecting rod, it is considered like .......... for buckling about X-axis.
   (a) both ends fixed
   (b) both ends hinged
   (c) one end fixed and the other end hinged
   (d) one end fixed and the other end free
7. Which of the following statement is wrong for a connecting rod?
   (a) The connecting rod will be equally strong in buckling about X-axis, if $I_{xx} = 4 I_{yy}$
   (b) If $I_{xx} > 4 I_{yy}$, the buckling will occur about Y-axis.
   (c) If $I_{xx} < 4 I_{yy}$, the buckling will occur about X-axis.
   (d) The most suitable section for the connecting rod is T-section.
8. The crankshaft in an internal combustion engine
   (a) is a disc which reciprocates in a cylinder
   (b) is used to retain the working fluid and to guide the piston
   (c) converts reciprocating motion of the piston into rotary motion and vice versa
   (d) none of the above
9. The rocker arm is used to actuate the inlet and exhaust valves motion as directed by the
   (a) cam and follower   (b) crank
   (c) crankshaft   (d) none of these
10. For high speed engines, a rocker arm of......... should be used.
    (a) rectangular section   (b) I-section
    (c) T-section   (d) circular

ANSWERS

1. (a)   2. (d)   3. (b)   4. (c)   5. (a)
6. (b)   7. (d)   8. (c)   9. (a)   10. (b)