10122ME603 DESIGN OF TRANSMISSION SYSTEMS **L T P C**
3 1 0 4

OBJECTIVE

To gain knowledge on the principles and procedure for the design of power Transmission components. To understand the standard procedure available for Design of Transmission sip terms To learn to use standard data and catalogues

UNIT I DESIGN OF TRANSMISSION SYSTEMS FOR FLEXIBLE ELEMENTS 9

Selection of V belts and pulleys-selection of Flat belts and pulleys-Wire ropes and pulleys – Selection of Transmission chains and Sprockets.Design of pulleys and sprockets.

UNIT II SPUR GEARS AND PARALLEL AXIS HELICAL GEARS 9

Gear Terminology-Speed ratios and number of teeth-Force analysis -Tooth stresses Dynamic effects - Fatigue strength - Factor of safety - Gear materials – Module and Face width-power rating calculations based on strength and wear considerations Parallel axis Helical Gears – Pressure angle in the normal and transverse plane-Equivalent number of teeth-forces and stresses. Estimating the size of the helical gears.

UNIT III BEVEL, WORM AND CROSS HELICAL GEARS 9

Straight bevel gear: Tooth terminology, tooth forces and stresses, equivalent number of teeth. Estimating the dimensions of pair of straight bevel gears. Worm Gear: Merits and demeritsterminology.Thermal capacity, materials-forces and stresses, efficiency, estimating the size ofthe worm gear pair. Cross helical: Terminology-helix angles-Estimating the size of the pair of
cross helical gears.

UNIT IV DESIGN OF GEAR BOXES 9

Geometric progression - Standard step ratio -Ray diagram, kinematics layout -Design of sliding
mesh gear box -Constant mesh gear box. – Design of multi speed gear box.

UNIT V DESIGN OF CAM CLUTCHES AND BRAKES 9

Cam Design: Types-pressure angle and under cutting base circle determination-forces and surface stresses. Design of plate clutches –axial clutches-cone clutches-internal expanding rim clutches-internal and external shoe brakes.

L:45 T: 15 TOTAL: 60

NOTE: (*Usage of P.S.G Design Data Book is permitted in the University examination*)

TEXT BOOKS:

1. Shigley J.E and Mischke C. R., “*Mechanical Engineering Design*”, Sixth Edition, Tata McGraw-Hill , 2003.
2. Sundararajamoorthy T. V, Shanmugam .N, "*Machine Design*", Anuradha Publications, Chennai, 2003.
3. Maitra G.M., Prasad L.V., “*Hand book of Mechanical Design*”, II Edition, Tata McGraw-Hill, 1985.
4. Bhandari, V.B., “*Design of Machine Elements*”, Tata McGraw-Hill Publishing Company Ltd., 1994.
5. Prabhu. T.J., “*Design of Transmission Elements*”, Mani Offset, Chennai, 2000,
6. Hamrock B.J., Jacobson B., Schmid S.R., “*Fundamentals of Machine Elements*”,

UNIT-1

UNITS, DIMENSIONS & CONVERSION FACTORS

Unit

All meaningful measurements in engineering science consist of at least two parts - a magnitude and a unit. Thus the measurement 3 metres consists of the magnitude or number, 3, and the unit, in this case the metre.

Entity

A unit is measure of a certain physical 'entity', for example the centimetre unit is a measure of the entity length, the kilometre/hour a unit of the entity speed, and so on.

Entity Inter-Relationship - Dimensions

For any particular physical system under review, it is convenient to class the entities which occur in it as either *fundamental* or *derived.* The fundamental entities are those which are selected - purely on convenience grounds - as basic buiding blocks of the system. The derived entities are then expressible as combinations of the fundamentals raised to certain powers - we say that the derived entities *'have certain dimensions'*in the fundamentals.

Suppose for example a geometric system was being examined - a system comprising the entities length (denoted by [L]), area [A], volume [V] and angle [φ]. Suppose also that length is chosen to be the fundamental entity, in which case area may be conceived as a length multiplied by a length - this is written dimensionally as $[A] = [L]*[L] = [L^2]$ - that is *the derived entity area has the dimensions of length squared.* Similarly $[V] = [A]*[L] = [L^3]$ and $[\varphi] = [L]/[L] = [L^0] = [\ ]$ since an angle may be thought of as the arc length of a circular sector divided by its radius. Entities in which the powers of the fundamentals are zero are termed *'dimensionless',* written [ ].

Alternatively, if volume had been selected as fundamental, then $[L] = [V^{1/3}]$, $[A] = [V^{2/3}]$ while $[\varphi]$ remains dimensionless in terms of any fundamental.

Homogeneity of Equations

The equation : 1 kilometre + 2 litres = 3 seconds is obviously meaningless, and is so because the terms are of different entities. An equation must be *entity homogeneous* - ie. the terms of the equation must all be of the same entity - before the arithmetic 1 + 2 = ? can be carried out.

Entity homogeneity is not always so obvious as in the above example. Suppose we set out to calculate the area 'A' of the Earth's surface which is visible from an aeroplane flying at height 'h' above the surface, the Earth being assumed spherical of radius 'r'. Suppose we came up with the expression : $A = 2\pi\, h\, r^3 / (r+h)$. Choosing length [L] as the fundamental entity, then the dimensions of the RHS of this equation are : $[L^0]*[L^1]*[L^3]/([L]+[L]) = [L^4]/[L^1] = [L^3]$ since 2π may be regarded either as a pure number or as an angle, both of which are dimensionless.

The LHS of the equation, being an area, has the dimensions $[L^2]$, so the dimensions of the two sides of the equation are different - the equation is not dimensionally homogeneous and must therefore be wrong - we can spot the error immediately. The necessity for entity homogeneity leads to a powerful technique known as *Dimensional Analysis* for actually setting up the form of the equations governing any physical system, as opposed to the mere checking above. We shall not examine the technique further here.

Consider now the equation : 1 centimetre + 2 metres = 3 kilometres. This too is obviously incorrect even although it is homogeneous in the entity length. It is wrong because the units of the various additive terms are not identical - the equation is not *unit homogeneous* and so $1 + 2 \neq 3$. The process of homogenising this simple equation will be second nature to the reader, however the underlying principles of homogenisation still apply, and must be spelt out in detail, when dealing with more complicated functions. The technique is outlined below.

Conversion Factors

Conversion factors are dimensionless numbers which inter-relate, or convert, different units of the same entity. Thus '100 centimetres/metre' and '60 second/minute' are familiar conversion factors - their dimensions are respectively [L]/[L] = [ ] and [T]/[T] = [ ] where 'T' is the entity time.

Considering the above equation : 1 cm + 2 m = ? again, the homogenising procedure is :

$$\begin{array}{cccccccccccccc} 1 & * & \frac{1}{10^2} & * & \frac{1}{10^3} & + & 2 & * & \frac{1}{10^3} & = & 10^{-5} & + & 2*10^{-3} & = & 2.01*10^{-3} \\ \cancel{\text{cm}} & & \frac{\cancel{\text{m}}}{\cancel{\text{cm}}} & & \frac{\text{km}}{\cancel{\text{m}}} & & \cancel{\text{m}} & & \frac{\text{km}}{\cancel{\text{m}}} & & \text{km} & & \text{km} & & \text{km} \end{array}$$

Note that before any addition is carried out, each term is brought to a common unit by successive application of conversion factors. Whether these multiply or divide is seen by inspection of the units - in the present case they are applied to eliminate 'cm' and 'm' in turn. The resulting common unit of the additive terms is 'km', though 'm' is much preferred as it eliminates any power of 10 - ie. the most succint answer is 2.01 m.

Introduction to Unit Systems

Although the Systeme International d'Unites (SI) is now firmly established as the standard system of units in this country, the US system - which is similar to the old Imperial System - is still very common in the mineral and resource processing fields,

and looks likely to remain so for some time. This, and the necessity to interpret overseas texts, means that engineers must be fluent in systems other than the SI. We shall examine the SI and the Imperial below, with a view to illustrating the techniques of converting from any one system to any other.
Three fundamental entities only are a necessary foundation for Mechanics (though Thermodynamics and Electricity each requires an additional fundamental). Those usually selected - on the basis of simplicity - are length [L], mass [M] and time [T], and their units are *defined arbitrarily.* The units of the derived entities are then based on these fundamental units.

Newton's Second Law
This law, which states that : force = mass * acceleration is the foundation of Mechanics, and must be homogeneous in both entities and units. Acceleration is 'time rate of change of velocity', that is a velocity increment divided by a time increment - velocity itself being displacement per unit time. So [acceleration] = ([L]/[T])/[T] = $[LT^{-2}]$ where L and T are fundamental. For entity homogeneity therefore, Newton's Law requires that the dimensions of the derived entity force, F, are given by : [F] = $[M]*[LT^{-2}] = [MLT^{-2}]$.
For unit homogeneity on the other hand : units of force = units of mass * units of length / (units of time)^2 and to ensure this the equation is written : force = mass * acceleration / g_c where g_c is a *dimensionless conversion factor* inserted to preserve unit homogeneity in whatever system is being used - that is each system is characterised by its own unique g_c.

It will be seen below that in some systems the magnitude of g_c equals the magnitude of standard gravitational acceleration - this results from the manner in which the unit of the derived entity force is defined in the system and is NOT due to any supposed direct dependence of g_c on gravity - g_c has got nothing directly to do with gravity - it is simply a conversion factor.

The Systeme International
In this system the standard units of the fundamental entities are defined as follows :

mass	- kilogram	(kg)	: the mass of an arbitrary lump of platinum
length	- metre	(m)	: an arbitrary number of wavelengths of atomic krypton radiation (nominally an even fraction of the Earth's equatorial circumference)
time	- second	(s)	: an arbitrary fraction of the year 1900

The unit of the derived entity force is called the Newton (N) and is defined as the force necessary to produce an acceleration of 1 m/s^2 in a mass of 1 kg. The corresponding value of g_c follows from the necessary homogeneity of Newton's Law :

$g_c = ma/F = 1 \{kg\} * 1 \{m/s^2\} / 1 \{N\} = 1\ kg.m/N.s^2$

The weight, W, of a mass of 5 kg for example, is the force exerted by the Earth on the mass, accelerating the mass if not equilibrated at the acceleration of gravity (for which the standard value is 9.81 m/s^2). So:

$W = ma/g_c = 5 \{kg\} * 9.81 \{m/s^2\} / 1 \{kg.m/N.s^2\} = 49\ N$

The reader is advised to follow through the cancelling of units here, in a manner similar to the '1+2=?' equation above. Systems like the SI in which the magnitude of g_c is unity are called 'absolute' systems.

The Imperial System

In the Imperial System the unit of mass, the 'pound' (lb), and the unit of length, the 'foot' (ft), are currently defined in terms of SI units - conversion factors of 2.205 lb/kg and 0.3048 m/ft apply - while the unit of time is identical to the SI second.

The unit of force, the 'pound force' (lbf), is defined as the weight of a pound mass under standard gravity conditions when the acceleration is 32.174 ft/s^2. The value of g_c for the Imperial system is found in a manner identical to the above :

$g_c = ma/F = 1\{lb\} * 32.174 \{ft/s^2\}/1 \{lbf\} = 32.174\ lb.ft/lbf.s^2$

The weight of a 5 lb mass under standard gravity conditions is therefore :

$W = ma/g_c = 5 \{lb\} * 32.174 \{ft/s^2\}/32.174 \{lb.ft/lbf.s^2\} = 5\ lbf$

Again, the reader should confirm manipulation of the units. Systems like the Imperial in which a mass's weight is equal numerically to the mass itself are called 'gravitational systems'. US notation differs from the Imperial in using the abbreviations 'lbm' for pound mass, and 'lb' for pound force.

Suppose it is required to find the weight of a body, W_M, at a point on the Moon where the gravitational acceleration is 5.2 ft/s^2, the body weighing 3 lbf under standard Earth gravity conditions. From the above, we see immediately that the mass of the body is 3 lb, so, using the last equation

$W_M = ma/g_c = 3\{lb\} * 5.2\{ft/s^2\}/32.174 \{lb.ft/lbf.s^2\} = 0.48\ lbf$

Conversion of force from one system to another is usually most easily carried out by invoking the g_c factors of the two systems. Thus to find the Imperial equivalent of 10 N, we have :

$$F = \frac{10 * 1 * 2.205}{0.3048 * 32.174} = 2.25\ lbf$$

$$\cancel{N} \quad \frac{\cancel{kg}.\cancel{m}}{\cancel{N}.s^2} \quad \frac{lb}{\cancel{kg}} \quad \frac{ft}{\cancel{m}} \quad \frac{lbf.s^2}{lb.ft}$$

Proceeding from the left, the derived unit in one system (eg. N here) is first reduced to fundamental units in that (SI) system, then inter -system fundamental conversion factors are used - in numerator or denominator *by inspection* - to convert to fundamental units in the second system (Imperial here) before final reduction to the derived units in the second system via that system's g_c. This approach is quite general and may be used for derived entities other than force.

Energy

All forms of energy - be they mechanical, thermal, electrical, nuclear etc. - are equivalent, that is they have the same dimensions. The two forms of most immediate interest to us are thermal energy or 'heat', Q, and mechanical energy or 'work', W (whose symbol should not be confused with that of weight). Their equivalence is expressed in the First Law of Thermodynamics : W = Q (briefly, this will be formulated more rigorously in Thermodynamics).

Work, a derived entity, is conceived as that which is done when a force moves its point of application, and is the product of the force and the distance moved in the direction of the force, so :

$$[\text{energy}] = [\text{force}] * [\text{distance}] = [MLT^{-2}] * [L] = [ML^2T^{-2}]$$

The unit of work, energy and quantity of heat in the SI is the 'Joule' { J } which is defined as the work done when a force of 1 N moves its point of application through a distance of 1 m - ie. a conversion factor of 1 J/Nm applies.
In the Imperial system, different units are used for thermal and mechanical energies, so a conversion factor '*J*' is necessary in the First Law statement, thus : W = *J*Q.

J is sometimes referred to as 'the mechanical equivalent of heat', however it's just another conversion factor - analogous to Newton's g_c but for the entity energy/work/heat. The Imperial unit of work is the foot-pound force { ft.lbf }, and the unit of heat is the British Thermal Unit { BTU }. Using these units, *J* has the value of 778 ft.lbf/BTU (approx). So, from the First Law, the amount of work which is equivalent to 10 BTU of heat is :

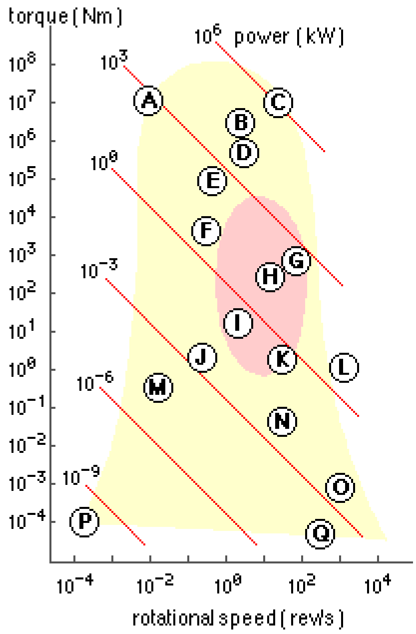
W = 778 { ft.lbf/BTU } * 10 { BTU } = 7780 ft.lbf

When inserting values into algebraic equations, the reader is strongly advised to adopt the technique used above, in which :

- the units associated with each number are written underneath the number
- a running check of the units is kept so that placement of conversion factors, in numerator or denominator, can quickly be determined.

V-BELT DRIVES

The great majority of mechanical power transmission applications involve rotating shafts, since rotation is continuous and the shafts / mountings are cheap relative to other means of power transmission. Matching a prime- mover to a load thus involves transformation of power between shafts - usually from a high speed / low torque drive shaft, through a speed reducer of ratio $R \geq 1$, to a low speed / high torque load shaft. So far, we have considered only electric motor prime movers, with industrial load speeds of the order of 10 Hz - however the range of torques and speeds encountered in practice is much wider than this viewpoint, as this diagram from Palmgren *op cit* suggests :-



A rotary cement kiln
B container ship gas turbine
C electric power generator
D diesel engine on ferry
E caterpillar
F windmill
G refrigeration compressor
H Volvo 340 automobile
I washing machine
J windscreen wiper motor
K hand power tool
L truck turbocharger
M timer clock
N electric razor
O gyroscope
P recording cylinder
Q dentist's drill

As has been noted, speed reducers are employed almost invariably to amplify torque rather than to reduce speed. The two most common speed reduction mechanisms in industry are belts (usually V-belts) and gears - though chains, hydrostatic transmissions or other drives may be used.

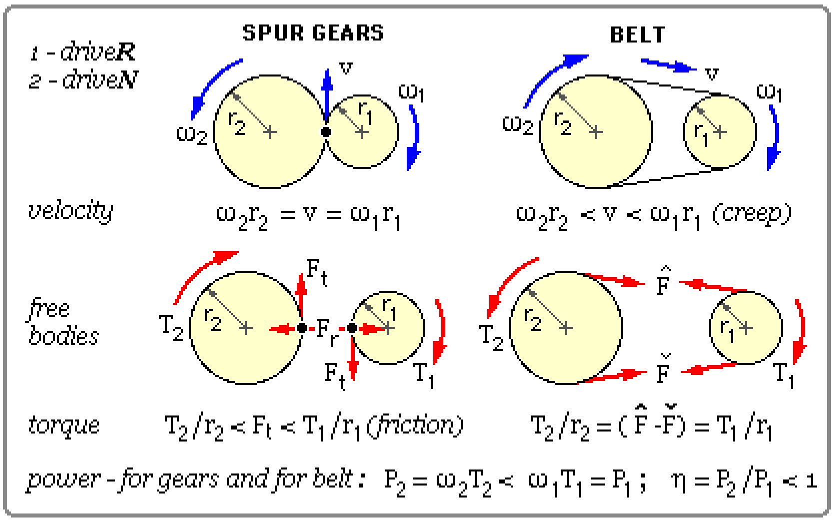
The factors other than cost which must be borne in mind when choosing a reducer are listed here - however shock absorption capacity, distance between shaft centres, accuracy required of shafts and mountings, tolerable vibration levels and so on may also need to be considered.

CHARACTERISTICS OF SPEED REDUCERS

	gears	chain	timing belt	V-belt	flat belt	hydrostatic trans.
drive mechanism	positive	positive	positive	friction	friction	-
max capacity, kW	10 000	500	500	1 000	1 000	1 000
max torque, Nm	10^8	10^6	10^4	10^4	10^4	10^8
max speed, m/s	50	15	60	30	50	-
speed ratio accuracy	high	high	high	moderate	moderate	moderate
full load efficiency, %	94-98	94-98	93-98	92-97	92-97	84-92
lubrication required	yes	yes	no	no	no	-

The efficiencies of belts are generally less than those of gears - that is why belts are not found in the main drive train of road vehicles where fuel economy is critical.

The diagram below compares the kinematics and kinetics of a pair of mating spur gears with those of a belt wrapped around two pulleys. The gears are represented by their pitch cylinders which roll without slip on one another, slip being prevented by the *positive drive* - ie. by the meshing teeth.



The transfer of power between gears is enabled by the normal action/reaction force at the tooth contact - or more particularly by the tangential component of this force, F_t, whose moment about the centre of each free body equilibrates the shaft torque T (assuming constant velocity). Friction plays only a minor role - inescapable and deleterious but subsidiary nonetheless.
The transfer of power in a belt drive on the other hand relies critically on friction. The tensions F_{min} & F_{max} in the two *strands* (the nominally straight parts of the belt not in contact with the pulleys) cause a normal pressure over the belt- pulley contact, and it is the corresponding distributed friction whose moment about the pulley centre equilibrates the shaft torque T - provided gross slip of the belt on the pulley surface does not occur due to friction breakaway.

Ideally, for gears and for belts, the speed reduction ratio and the torque amplification ratio are each equal to the radius ratio, so that the output power equals the input power and the efficiency is 100%. The speed ratio across a real pair of gears always equals the ideal ratio because of the positive drive, however sliding friction results in a torque ratio which is less than ideal. A real belt drive is just the opposite - the torque ratio equals the ideal ratio (as may be seen from the free bodies), but *creep* results in the speed ratio being less than ideal. Creep - not to be confused with gross slip - is due to belt elements changing length as they travel between F_{min} & F_{max}, and since the pulley is rigid then there must be relative motion between belt element and pulley.
Since power equals the product of torque and (angular) speed, the consequence of the foregoing is that efficiencies of real gears and belts are less than 100%.

Some of the many forms of belt are introduced below.

Historically, *flat belts* made from joined hides were first on the scene, however modern flat belts are of composite construction with cord reinforcement. They are particularly suitable for high speeds.

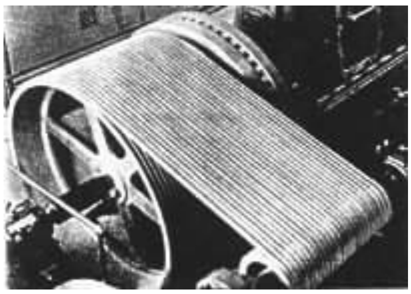
Classical banded (ie. covered) *V-belts* comprise cord tensile members located at the pitchline, embedded in a relatively soft matrix which is encased in a wear resistant cover. The wedging action of a V-belt in a pulley groove results in a drive which is more compact than a flat belt drive, but short centre V-belt drives are not conducive to shock absorption.

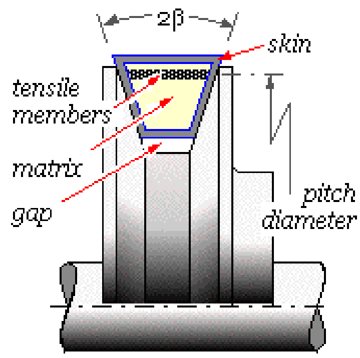
Wedge belts are narrower and thus lighter than V-belts. Centrifugal effects which reduce belt-pulley contact pressure and hence frictional torque are therefore not so deleterious in wedge belt drives as they are in V-belt drives.

Modern materials allow *cut* belts to dispense with a separate cover. The belt illustrated also incorporates slots on the underside known as *cogging* which alleviate deleterious bending stresses as the belt is forced to conform to pulley curvature. Cogging should not be confused with the teeth on . . . .

Synchronous or *timing belt* drives are positive rather than friction drives as they rely on gear- like teeth on pulley and belt enabled by modern materials and manufactureing methods. They are mentioned here only for completeness - we shall not examine them further.

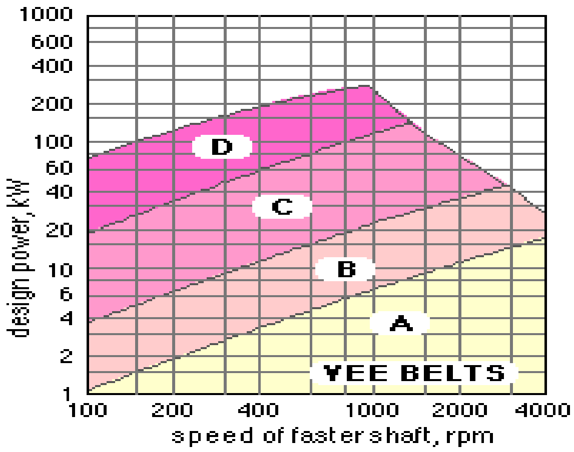
If a single V-belt is inadequate for power transmission then multiple belts and corresponding multi- grooved pulleys are necessary - this pulley is equipped with a tapered bush for axle clamping without the stress concentration associated with a key. The rather extreme short-centre drive on the left illustrates a problem with multiple belts - how to ensure equitable load sharing between flexible belts whose as-manufactured dimensional tolerances are significanty looser than those of machined components for example. Two types of belt for avoiding mismatched lengths are shown :

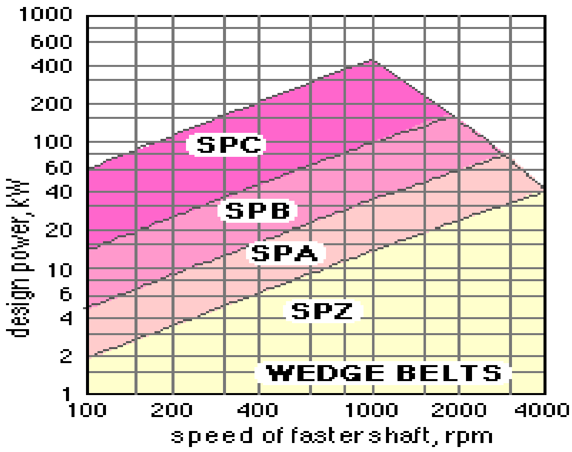




Each component of a V-belt performs a particular function. The main load- carrying elements are the tensile members, often in the form of longitudinally stiff rayon cords located near the centroidal axis of the belt's cross-section, embedded in a relatively soft elastomeric matrix whose main purpose is to channel the load from the contacts with the groove sides into the tensile members.

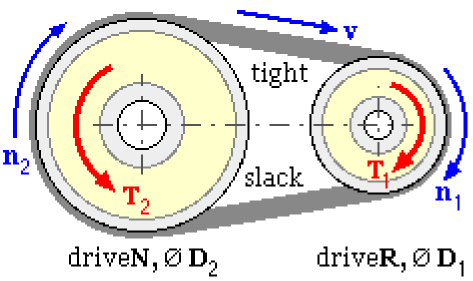
The groove semi-angle lies usually in the range $17^o \leq \beta \leq 19^o$. It should be noted that there is a gap ie. no contact at the bottom of the groove. Flat belts may be regarded as particular cases of V-belts in which $\beta = 90^o$.





V-belts are available in a number of standard cross-sectional sizes, designated in order of increasing size A, B, etc, while wedge belts are designated variously as SPA, SPB, etc (or α, β etc in the US). Each size is suitable for a particular power range as suggested by the carpet diagrams. The regions of applicability for the various sizes in these diagrams overlap substantially.

As the belts are endless, only certain discrete standard pitch lengths are manufactured. The power demand very often necessitates a number of matched belts on multi-grooved pulleys, as illustrated above.
Discrete dimensions apply also to off- the- shelf pulleys, which are available only with certain recommended pitch diameters and number of grooves. A special pulley may be manufactured of course - but would cost more than a mass- produced commercial product. A pulley is referred to by its pitch diameter - other dimensions including its OD are available from suppliers' manuals which should be consulted also for local availability.

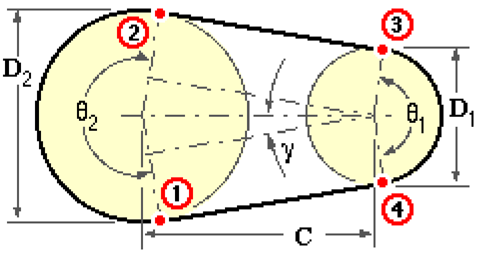


A typical V-belt drive is illustrated. The effective (or pitch) diameter of the small driveR pulley is D_1 ; that of the large driveN pulley is D_2 .
Before the drive starts to rotate and transmit power, an initial tension F_o is produced in both belt strands by the shafts being pulled apart and then locked (eg. by a motor on slide rails or by other means).
Drive commences by the power source applying a clockwise (say) torque T_1 to the shaft of the small driveR pulley, causing it to rotate clockwise at a steady speed n_1 (rev/s). The tension in the *'tight'* upper straight strand will then exceed F_o while the tension in the *'slack'* lower strand will become less than F_o - this tension difference applies a torque to the driveN load pulley, equilibrating the load torque T_2 while the pulley rotates at uniform speed n_2 , also clockwise. Neglecting creep:

(1) $v = \pi D_1 n_1 = \pi D_2 n_2$; speed ratio, $R = n_1 / n_2 = D_2 / D_1 \geq 1$
where the belt speed, v, is limited to 30 m/s for the usual cast iron pulley material, though higher speeds can be achieved with more expensive builds. V-belts are designed for optimum performance at speeds of around 20 m/s.

In the most common V-belt drive design problem, the transmitted power and the speed of the small pulley are stipulated, together with a specified range of the speed ratio and possibly acceptable ranges of shaft centre distance and of drive life - though if the drive is designed by the methods outlined by the Code(s) then its life, though presumably commercially acceptable, cannot be evaluated.
The required drive is specified by a suitable size (B, or γ, etc), number and length of the belt(s), and by the two pulley diameters. We therefore consider two aspects of V-belt drives :

- the drive overall geometry, ie. the inter-relationship between centre distance and belt length
- the fatigue life of the belts as dictated by the loading on them, which is in turn affected by the drive kinetics and the power transmitted through the drive.



Overall geometry

V-belt drives are essentially short centre drives. If in drive design the centre distance C is not specified, then it should be set at around $2D_1 \sqrt{(R+1)}$ but preferably not less than D_2 . Since the diameters and belt length are discrete variables so also is the theoretical centre distance, though in the absence of idlers the nominally fixed centre distance must be capable of slight variation by motor slide rails for example, to allow for belt installation and subsequent take-up (initial tightening) before rotation commences. This capability also allows for manufacturing tolerances on belt length, L. From the geometry :-

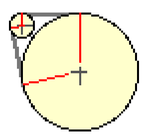
(2a) $L = {}^{\pi}/_{2} (D_1 + D_2) + 2C (\gamma \sin\gamma + \cos\gamma)$; where $\sin\gamma = (D_2 - D_1)/2C$

This is used to find the belt length, L, for given centre distance, C (and pulley diameters). Conversely, to find the centre distance corresponding to a certain belt length, (2a) must be solved iteratively - a very close first approximation is given by :-

(2b) $4\,C = \sqrt{[\, g^2 - 2(D_2 - D_1)^2 \,]} + g$; where $g = L - {}^{\pi}/_{2} (D_1 + D_2)$

The *wrap angle* (or "arc of contact") on each pulley is evidently :-

(2c) $\theta_1 = \pi - 2\gamma$; $\theta_2 = \pi + 2\gamma$; $\theta_1 \leq \theta_2$



The power transmission capability of a drive is usually limited by the arc of contact on the small pulley, and so is reduced by large speed ratios and by short centres - eg. θ_1 is only about 100^o here.

Any particular cross-section of the belt traverses alternately the slack and tight strands and is subject to bending when in way of one of the pulleys, so it is clear that cyclic loading and fatigue are prevalent. Before we can look at fatigue however, we have to know the belt forces and stresses. These will depend on the belt load and speed - so let us now consider the belt kinetics

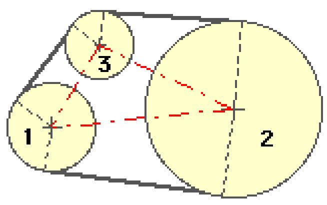
1. A belt drive incorporates a small pulley of 100 mm diameter and a belt whose length is 1100 mm. For speed ratios of (a) 1.5:1 (b) 2:1 (c) 3.15:1, calculate the theoretical shaft centre distance and angle of wrap on the small pulley.
   [353, 310, 193 mm]

2. (a) Use the belt properties of Table 1 to calculate the basic rating of an A-section V-belt with two 100 mm diameter pulleys rotating at 4200 rpm. Check, using the Code tables. [3.07 kW]
   (b) If the pitch length of the above belt is 3080 mm, what then is the rating and what is the corresponding power correction factor for pitch length? Check this last value with the Code. [3.50 kW, 1.14]
   (c) If the drive in (a) is required to last for only 10 kh, by what percentage is the above capacity increased? [24%]

3. Plot rating versus belt speed, similar to the above rating curves, for an A-section belt. Use pulley diameters of 75, 132, 250, 500 and 1000 mm. Superimpose upon this, trajectories of constant effectiveness : 20, 40, 60, 80 and 100%. Comment upon the effect of pulley diameter on rating as the diameter increases.

4. A V-belt drive employs a single B belt of length 2300 mm, together with 200 and 400 mm diameter pulleys. The smaller pulley rotates at 1440 rpm.
   (a) What is the capacity of this drive using the Code method ? [6.04 kW]
   (b) What is the life of the drive when transmitting 6.04 kW ? [31 kh]
   (c) Repeat (a), but use (5a) with the standard life of 26 kh. [6.19 kW]
   (d) Check this last result using the program *V-belts.*
   (e) A multi-strand drive, otherwise identical to the above, is required to transmit 12 kW with a duty factor of 1.3. Use (5a) to determine the number of belts required. [2.5]
   (f) What life may be expected, if 2, or if 3 belts are used ? [5.0, 82 kh]

5. Two 1750 mm long A-section belts are incorporated into the drive whose layout is sketched. The wrap angle on



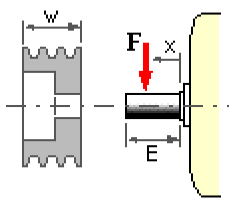
the 150 mm diameter motor pulley (1) is 118^{o} and the pulley rotates at 2880 rpm. The 400 mm diameter driven pulley (2) absorbs the design power of 10 kW. Pulley (3), of 80 mm diameter, is an idler and absorbs no appreciable power.
Estimate the life of the belts if . . . . .
(a) the pulleys rotate clockwise, or
(b) they rotate counterclockwise, or
(c) the idler is removed and the centre distance between (1) and (2) increased accordingly. [6, 0.6, 14 kh]

6. A 7.5 kW 1445 rpm squirrel cage motor, started direct-on-line, is required to drive a machine tool at a speed of around 860 to 870 rpm. Duty is expected to be 7 hr/day, 5 days/week, 49 weeks/year with 4 years between belt replacements. The centre distance should lie within the range 280 to 320mm.
Select a drive for this duty.

7. Select a suitable hinge location for the pivoted motor drive of the foregoing worked example in which an ABB MBT 132M motor transmits 7 kW to a launderer through three B2500 belts on 180 and 514 mm diameter pulleys, the motor lying vertically under the launderer pulley.

8. A blower absorbs 3.5 kW at its design speed of 650 rpm, and is equipped with a 260 mm diameter, 90 mm wide flat pulley. It is proposed to drive it by a pivoted motor, V-flat arrangement.
Select a squirrel cage motor and finalise the drive, including pivot location.

9. A squirrel cage motor is usually equipped with deep groove ball bearings, but life considerations might require replacement of the drive-end bearing by a larger capacity roller bearing when :
- the shaft load is heavy due to a small belt pulley for example, or
- the load overhang ('x' in the diagram) is large.
An ABB motor t

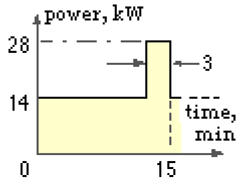


10.ype M2BA 280 SMA delivers 75 kW at 1485 rpm via a fully loaded belt drive comprising 5 SPB 3150 belts on 212 and 630 mm diameter pulleys at 889 mm centres. From catalogues *[ABB, Fenner]* the pulley width w = 102 mm, the motor shaft length E = 140 mm, and shaft radial loads F corresponding to two

life (kh)	40	63
F_0 at x=0 (kN)	7.32	6.29
F_E at x=E (kN)	6.18	5.31

different bearing lives at two load positions along the shaft are as tabulated :- Will the drive-end ball bearing last for the target life of 25 kh or is a roller bearing needed?

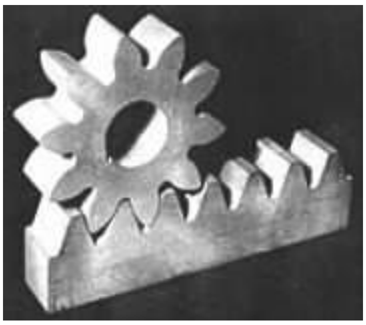
11. This concerns part-load belt tensions and components with different load-life equations.
Select a squirrel cage motor and fixed centre belts to drive an agitator at about 700 rpm for 1 kh per annum. The power demand varies cyclically as shown and a 3-year belt replacement period is acceptable.
The drive should be compact, but not to the extent that the motor's usual ball bearings have to be replaced by roller bearings.
For the purposes of this problem, motors' maximum shaft loads tabulated in the Motors chapter refer to loads at the end of the shaft (F_E of the previous problem), to bearing lives of 40 kh, and to a load-life index of n = 2.5.



UNIT-2
SPUR GEARS

Gears are used to transmit power between shafts rotating usually at different speeds. Some of the many types of gears are illustrated below.

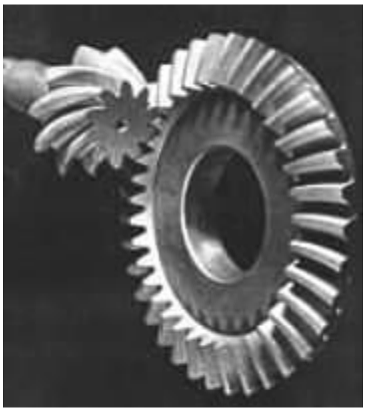
A pair of *spur* gears for mounting on parallel shafts. The 10 teeth of the smaller *pinion* and the 20 teeth of the *wheel* lie parallel to the shaft axes



A *rack* and pinion. The straight rack translates rectilinearly and may be regarded as part of a wheel of infinite diameter

Like spur gears *helical* gears connect parallel shafts, however the teeth are not parallel to the shaft axes but lie along helices about the axes

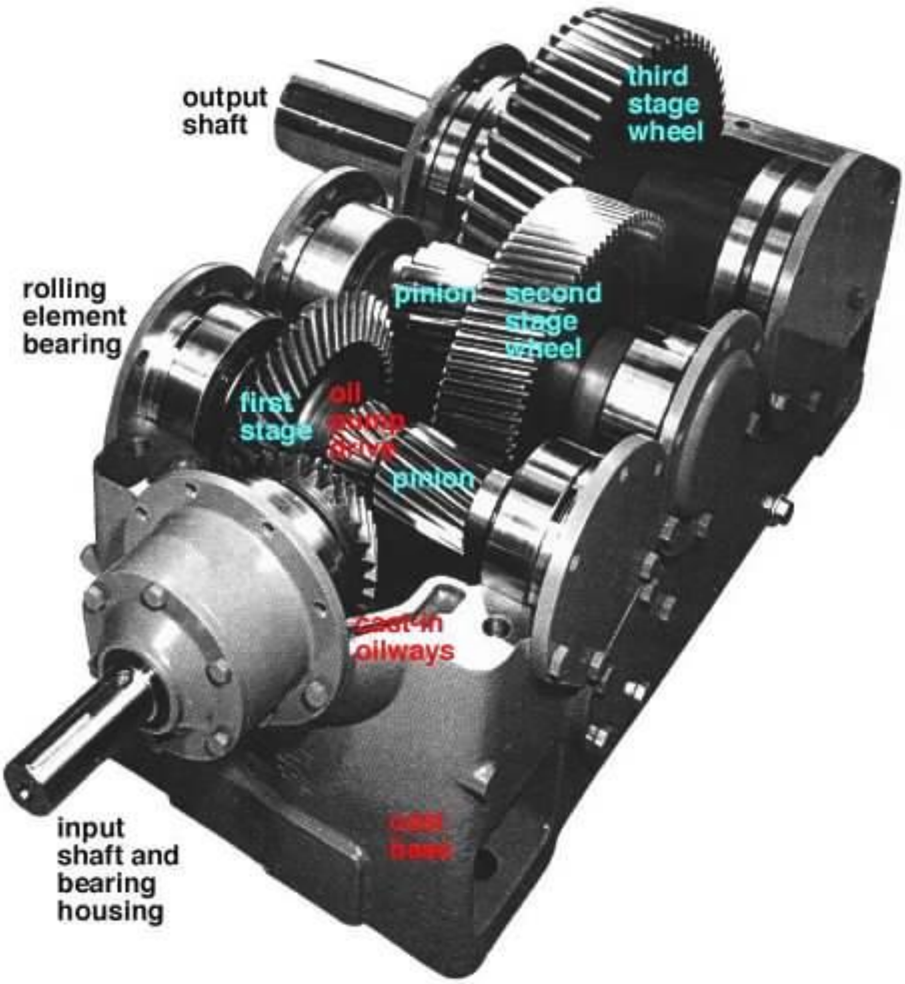
Straight *bevel* gears for shafts whose axes intersect



Hypoid gears - one of a number of gear types for offset shafts

A *worm* and wormwheel gives a large speed ratio but with significant sliding

In order to demonstrate briefly the development of gear drives, from first principles through to safety implications, we consider here only spur gears. Knowledge of these is fundamental to understanding the behaviour of geometrically more complex types, including helical gears which are generally preferred to spurs since they are more compact and smoother in operation, thus permitting higher speeds.



A typical commercial gearbox is shown with its cover removed. It demonstrates that it is usually more attractive economically to split a larger speed ratio into a number of *stages* (pairs of gears) rather than to effect it with a single pair. There are three stages here - the first spiral bevel pair is followed by two helical pairs.

A couple of features of the box are immediately apparent :

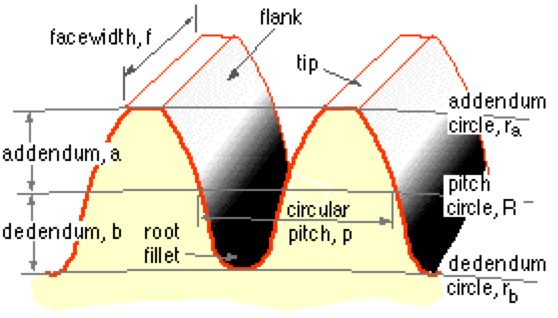
- compactness - shafts are short and simply supported where practicable, with gears located as close as possible to bearings in order to minimise shaft bending
- sturdiness increasing from input through output - the sizes of input & output shafts, and second & third stage gear teeth, should be compared.

A pair of meshing gears is a power transformer, a coupler or interface which marries the *speed and torque*characteristics of a power *source* and a power *sink* (load). A single pair may be inadequate for certain sources and loads, in which case more complex combinations such as the above gearbox, known as *gear trains*, are necessary. In the vast majority of applications such a device acts as a speed reducerin which the power source drives the device through the high speed low torque input shaft, while power is fed from the device to the load through the low speed high torque output shaft.

Speed reducers are much more common than speed -up drives not so much because they reduce speed, but rather because they amplify torque. Thus gears are used to accelerate a car from rest, not to provide the initial low speeds (which could be accomplished by easing up on the accelerator pedal) but to increase the torque at the wheels which is necessary to accelerate the vehicle. Torque amplification is the reason for the gearbox's increasing sturdiness mentioned above.

These notes will consider the following aspects of spur gearing :-

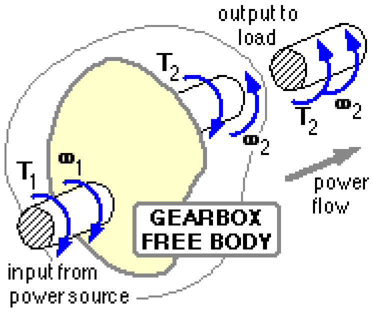
- overall kinetics of a gear pair (for cases only of steady speeds and loads)
- tooth geometry requirements for a constant velocity ratio (eg. size and conjugate action)
- detailed geometry of the involute tooth and meshing gears
- the consequences of power transfer on the fatigue life of the components, and hence
- the essentials of gear design.



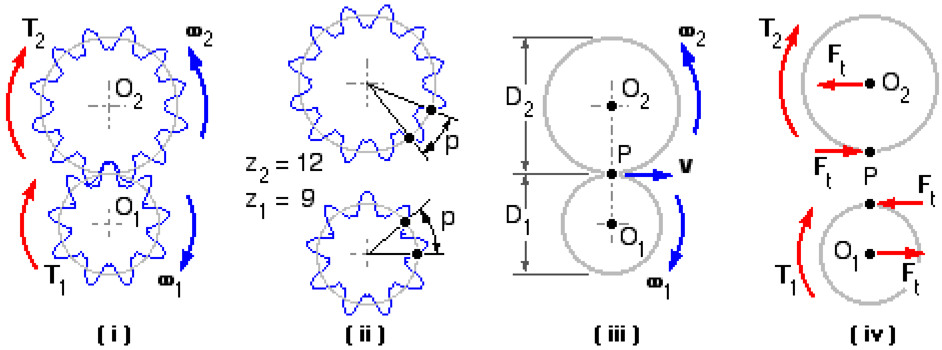
Some of the main features of spur gear teeth are illustrated. The teeth extend from the root, or *dedendum* cylinder (or colloquially,*"circle"*) to the tip, or *addendum* circle: both these circles can be measured. The useful portion of the tooth is the *flank* (or face), it is this surface which contacts the mating gear. The *fillet* in the root region is kinematically irrelevant since there is no contact there, but it is important insofar as fatigue is concerned.

Overall kinetics of a gear pair

Analysis of gears follows along familiar lines in that we consider kinetics of the overall assembly first, before examining internal details such as individual gear teeth. The free body of a typical single stage gearbox is shown. The power source applies the torque T_1 to the input shaft, driving it at speed ω_1 in the sense of the torque (clockwise here). For a single pair of gears the output shaft rotates at speed ω_2 in the opposite sense to the input shaft, and the torque T_2supplied by the gearbox drives the load in the sense of ω_2. The reaction to this latter torque is shown on the free body of the gearbox - apparently the output torque T_2 must act on the gearbox *in the same sense* as that of the input torque T_1.



The gears appear in more detail in Fig (i) below. O_1 and O_2 are the centres of the pinion and wheel respectively. We may regard the gears as equivalent *pitch cylinders* which roll together without slip - the requirements for preventing slip due to the *positive drive* provided by the meshing teeth is examined below. Unlike the addendum and dedendum cylinders, pitch cylinders cannot be measured directly; they are notional and must be inferred from other measurements.



One essential for correct meshing of the gears is that the *size* of the teeth on the pinion is the same as the size of teeth on the wheel. One measure of size is the *circular pitch*, p, the distance between adjacent teeth around the pitch circle (ii); thus $p = \pi D/z$ where z is the number of teeth on a gear of pitch diameter D. The SI measure of size is the *module*, $m = p/\pi$ - which should not be confused with the SI abbreviation for metre. So the geometry of pinion 1 and wheel 2 must be such that :

$$D_1 / z_1 = D_2 / z_2 = p / \pi = m$$

. . . . that is the module must be common to both gears. For the rack illustrated above, both the diameter and tooth number tend to infinity, but their quotient remains the finite module.

The pitch circles contact one another at the *pitch point*, P Fig (iii), which is also notional. Since the positive drive precludes slip between the pitch cylinders, the pinion's pitch line velocity, v, must be identical to the wheel's pitch line velocity :

$$v = \omega_1 R_1 = \omega_2 R_2 \quad ; \quad \text{where pitch circle radius} \quad R = D/2$$

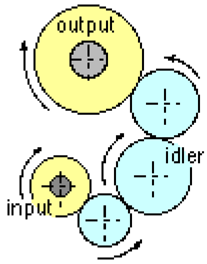
Separate free bodies of pinion and wheel appear in (iv).
F_t is the tangential component of action -reaction at the pitch point due to contact between the gears. The corresponding radial component plays no part in power transfer and is therefore not shown on the bodies. Ideal gears only are considered initially, so the friction force due to sliding contact is omitted also. The free bodies show that the magnitude of the shaft reactions must be F_t, and that for equilibrium :

$$F_t = T_1 / R_1 = T_2 / R_2 \quad \text{in the absence of friction.}$$

The preceding concepts may be combined conveniently into :-

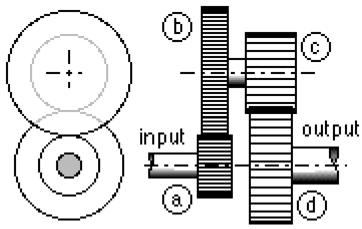
$$(1) \qquad \omega_1 / \omega_2 = T_2 / T_1 = D_2 / D_1 = z_2 / z_1 \quad ; \quad D = mz$$

That is, gears reduce speed and amplify torque in proportion to their teeth numbers.
In practice, rotational speed is described by N (rev/min or Hz) rather than by ω (rad/s).



The only way that the input and output shafts of a gear pair can be made to rotate in the same sense is by interposition of an odd number of intermediate gears as shown - these do not affect the speed ratio between input and output shafts. Such a gear train is called a *simple* train. If there is no power flow through the shaft of an intermediate gear then it is an *idler* gear.

A gear train comprising two or more pairs is termed *compound* when the wheel of one stage is mounted on the same shaft as the pinion of the next stage. A compound train as in the above gearbox is used when the desired speed ratio cannot be achieved economically by a single pair. Applying (1) to each stage in turn, the overall speed ratio for a compound train is found to be the product of the speed ratios for the individual stages.



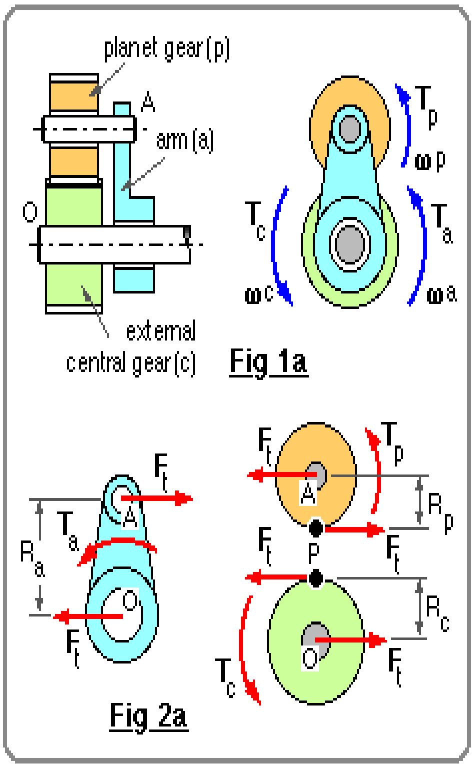
Selecting suitable integral tooth numbers to provide a specified speed ratio can be awkward if the speed tolerance is tight and the range of available tooth numbers is limited. Until the advent of computers allowed such problems to be solved by iterative trials, techniques based on *continued fractions* were used. Appendix A is provided to illustrate the concepts and advantages of continued fractions and attendant Padé approximations - this is for general interest, not just for gears.

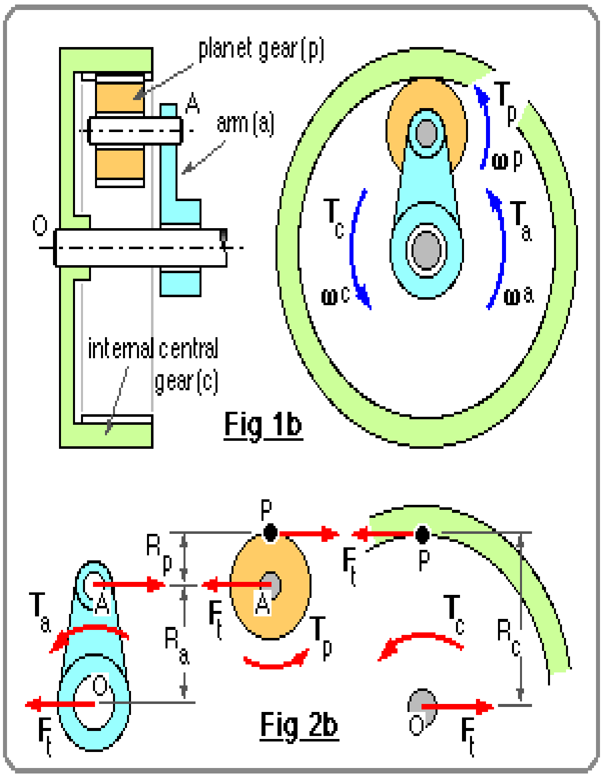
Unlike the above gearbox, the input and output shafts are coaxial in the train illustrated here; this is rather an unusual feature, but necessary in certain change speed boxes and the like.

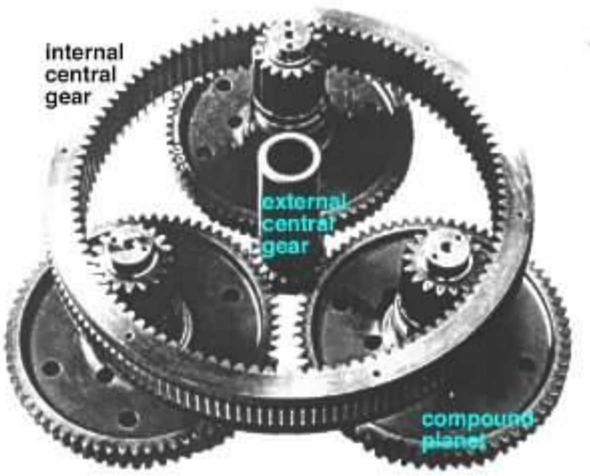
Epicyclic gear trains

An epicyclic train is often suitable when a large torque/speed ratio is required in a compact envelope. It is made up of a number of *elements* which are interconnected to form the train. Each element consists of the three *components* illustrated below :

- a *central gear* (c) which rotates at angular velocity ω_c about the fixed axis O-O of the element, under the action of the torque T_c applied to the central gear's integral shaft; this central gear may be either an external gear (also referred to as a *sun* gear) Fig 1a, or an internal gear, Fig 1b
- an *arm* (a) which rotates at angular velocity ω_a about the same O-O axis under the action of the torque, T_a - an axle A rigidly attached to the end of the arm carries
- a *planet gear* (p) which rotates freely on the axle A at angular velocity ω_p, meshing with the central gear at the pitch point P - the torque T_p acts on the planet gear itself, not on its axle, A.







The epicyclic gear photographed here without its arms consists of two elements. The central gear of one element is an external gear; the central gear of the other element is an internal gear. The three identical planets of one element are compounded with (joined to) those of the second element.

We shall examine first the angular velocities and torques in a single three-component element as they relate to the tooth numbers of central and planet gears, z_c and z_p respectively. The kinetic relations for a complete epicyclic train consisting of two or more elements may then be deduced easily by combining appropriately the relations for the individual elements.

All angular velocities, ω, are absolute and constant, and the torques, T, are external to the three-component element; for convenience all these variables are taken positive in one particular sense, say anticlockwise as here. Friction is presumed negligible, ie. the system is ideal.

Separate free bodies of each of the three components - including the torques which are applied one to each component - are illustrated in Figs 2a and 2b for the external and internal central gear arrangements respectively. Also shown are the shaft centre O and axle A, the radii R_c& R_p of the central and planet pitch cylinders, the radius of the arm R_a.

There are two contacts between the components :

- the planet engages with the central gear at the pitch point P where the action / reaction due to tooth contact is the tangential force F_t, the radial component being irrelevant;
- the free rotary contact between planet gear and axle A requires a radial force action / reaction; the magnitude of this force at A must also be F_t as sketched, for equilibrium of the planet.

With velocities taken to be positive leftwards for example, we have for the external central gear :

- geometry from Fig 2a : $R_a = R_c + R_p$
- velocity of P : $v_P = v_A + v_{PA}$ so with the given senses : $\omega_c R_c = \omega_a R_a - \omega_p R_p$
- torques from Fig 2a : $F_t = -T_c / R_c = -T_p / R_p = T_a / R_a$

and for the internal central gear :

- geometry from Fig 2a : $R_a \;=\; R_c - R_p$
- velocity of P : $v_P \;=\; v_A + v_{PA}$ so with the given senses : $\omega_c R_c \;=\; \omega_a R_a + \omega_p R_p$
- torques from Fig 2a : $F_t \;=\; -T_c / R_c \;=\; T_p / R_p \;=\; T_a / R_a$

Substituting for R_a from the geometric equations into the respective velocity and torque equations, and noting that $R_c/R_p \;=\; z_c/z_p$, leads to the same result for both internal and external central gear arrangements. These are the desired relations for the three-component element :

(2a) $\quad (\omega_c - \omega_a) z_c + (\omega_p - \omega_a) z_p \;=\; 0$
(2b) $\quad T_c / z_c \;=\; T_p / z_p \;=\; -T_a / (z_c + z_p)$

. . . . in which z_c is taken to be a positive integer for an external central gear, and a negative integer for an internal central gear.

It is apparent that the element has one degree of kinetic (torque) freedom since only one of the three torques may be arbitrarily defined, the other two following from the two equations (2b). On the other hand the element possesses two degrees of kinematic freedom, as any two of the three velocities may be arbitrarily chosen, the third being dictated by the single equation (2a).

From (2b) the net external torque on the three-component element as a whole is :

$$\Sigma T \;=\; T_c + T_p + T_a \;=\; T_c \{ 1 + z_p / z_c - (z_c + z_p)/z_c \} \;=\; 0$$

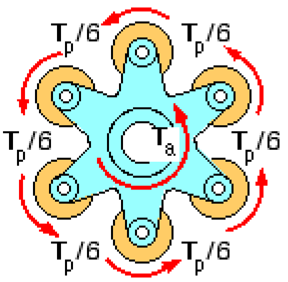
which indicates that equilibrium of the element is assured.

Energy is supplied to the element through any component whose torque and velocity senses are identical. From (2) the total external power being fed into the three-component element is :

$$\Sigma P \;=\; P_c + P_p + P_a \;=\; \omega_c T_c + \omega_p T_p + \omega_a T_a \;=\; T_c \{ \omega_c + \omega_p z_p /z_c - \omega_a (z_c + z_p)/z_c \}$$

$$= \; T_c \{ (\omega_c - \omega_a) z_c + (\omega_p - \omega_a) z_p \} / z_c \;=\; 0$$

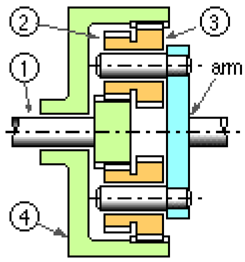
confirming that energy is conserved in the ideal element.



In practice, a number of identical planets are employed for balance and shaft load minimisation. Since (2) deal only with effects external to the element, this multiplicity of planets is analytically irrelevant provided T_p is interpreted as being the total torque on all the planets, which is shared equally between them as suggested by the sketch here. The reason for the *sun- and- planet* terminology is obvious; the arm is often referred to as the *spider* or *planet carrier*.

Application of the element relations to a complete train is carried out as shown in the example which follows. More complex epicyclic trains may be analysed in a similar manner, but the technique is not of much assistance when the problem is one of gear train design - the interested designer is referred to the Bibliography.

EXAMPLE



An epicyclic train consists of two three-component elements of the kind examined above. The first element comprises the external sun gear 1 and planet 2; the second comprises the planet 3 and internal ring gear 4. The planets 2 and 3 are compounded together on the common arm axles.

Determine the relationships between the kinetic variables external to the train in terms of the tooth numbers z_1, z_2, z_3 & z_4.

The train is analysed via equations (2) applied to the two elements in turn, together with the appropriate equations which set out the velocity and torque constraints across the interface between the two elements 1-2-arm and 3-4-arm.

1-2-arm :

$(\omega_1 - \omega_a) z_1 + (\omega_2 - \omega_a) z_2 = 0$ from (2a)

$T_1 / z_1 = T_2 / z_2 = - T_{a2} / (z_1 + z_2)$ from (2b)

3-4-arm :

$(\omega_4 - \omega_a) (-z_4) + (\omega_3 - \omega_a) z_3 = 0$ in which z_4 is a positive integer as

$T_4 / (- z_4) = T_3 / z_3 = - T_{a3} / (- z_4 + z_3)$ central gear is internal

T_{a2} and T_{a3} are the parts of the total external torque on the arm, T_a, which are applied individually to the two elements 1-2-arm and 3-4-arm.

Interface :

$\omega_3 = \omega_2$ since the planets 2 & 3 are coupled

$T_3 = - T_2$ since the planets 2 & 3 are coupled (action/reaction)

$T_a = T_{a2} + T_{a3}$ as the arm is common to both elements 1-2-arm and 3-4-arm

Solution :

The *basic speed ratio*, i_o, of an epicyclic train is defined as the ratio of input to output

speeds when the arm is held stationary.
Neither input nor output is defined here - indeed this terminology can be confusing with multiple degrees of freedom - so for example select gear 1 as input, gear 4 as output.
It follows that $i_o = (\omega_1 / \omega_4)_{\omega a=0}$.
Solving the three velocity equations and the six torque equations leads to the desired relations :

Velocities : $(\omega_1 - \omega_a) = i_o(\omega_4 - \omega_a)$ where $i_o = - z_2 z_4 / z_1 z_3$
Torques : $T_1 = -T_4 / i_o = T_a / (i_o - 1)$

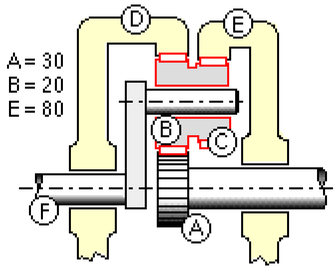
Evidently this train possesses the same degrees of freedom as an individual element.

Spur gear problems

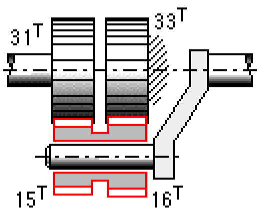
In the following problems, assume that :

- *gears with any tooth number up to 120 are procurable (constraints are more severe in practice)*
- *all gears are of steel, to the 20° full depth system unless otherwise indicated*
- *mid-range profile shifts apply, where relevant.*

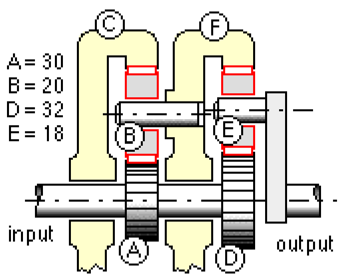
The program Steel Spur Gears should be used to assist solution of asterisked problems, and may be used to check longhand solution of other fatigue problems.



1. Tooth numbers of certain gears in the epicyclic train are indicated; all gears are of the same module. Gear A rotates at 1000 rev/min clockwise while E rotates anticlockwise at 500 rev/min.
   Determine the speed and direction of rotation of the ring-gear D and of the arm shaft F. If the power output through each of D and F is 1 kW, what are the power transfers through A and E?
   [371 rev/min anticlockwise; 40 rev/min clockwise; 8.77 kW input; 6.77 kW output]



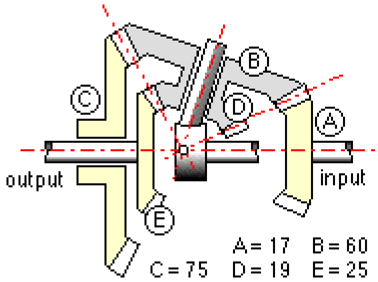
2. The arm of the epicyclic train is driven clockwise at 1450 rev/min by a 5 kW motor. What torque is necessary to lock the 33 teeth gear? What is the speed of the 31 tooth gear? *Note the reduction !*
   [16.3 kNm clockwise, 2.92 rev/min clockwise]



3. The sun wheels A and D are integral with the input shaft of the compound epicyclic gear illustrated, and

the annular wheel C is fixed. The planet wheel B rotates freely on an axle carried by the annular wheel F, and the planet E on an axle mounted on the output shaft's arm. Given the tooth numbers indicated, find the speed of the output shaft when the input shaft rotates at 1000 rev/min.
[524 rev/min]

4. In the epicyclic train illustrated, the gear C is fixed and the compound planet BD revolves freely on a spindle which is coaxial with the input and output shafts.
(a) Show that if $z_b z_e > z_c z_d$ then input and output shafts rotate in the same direction.
(b) 7.5 kW is fed into the input shaft at 500 rev/min, losses are negligible, and tooth numbers are sketched. Determine the torque on the output shaft.
[15.5kNm]



5. Select spur gears suitable for speed ratios of (i) $1/\sqrt{2}$, and (ii) π, to four significant figures.

6. Determine the practical limits of profile shift on a 6 mm module gear with 19 teeth. If a profile shift of 0.4 is implemented, what are the dedendum, base, pitch, extended pitch and addendum circle diameters of the gear ?
Evaluate the base pitch and the angle γ of Fig A.
[103.8, 107.1, 114, 118.8 and 130.8 mm. 17.7 mm, 6.47°]

7. Derive equation (10) from which the contact ratio may be calculated (20° full depth system).

8. What is the practical range of centre distance for a pair of 4 mm module spur gears with 19 and 35 teeth ? If they are manufactured with profile shifts of 1.5 mm and 2 mm respectively, evaluate the extended pressure angle and the contact ratio.
[$108.6 \leq C \leq 112.8$ mm, 24.47°, 1.42]

9. Use the design procedure outlined in the Notes to determine gears suitable for a speed ratio of $\sqrt{2} \pm 0.5$ % and a centre distance of 200 ± 1 mm.
[6 mm module, with 27 and 38 teeth, and profile shifts of 0.45 say, for pinion and correspondingly 0.38 for wheel]

10. Evaluate the contact ratio and the fatigue geometric factors I and J for each of the following :
(a) the pairs 13:35, 23:62 and 36:97 (which approximate the ratio 0.3711 to within 0.1%);
(b)* 23:62 teeth, assuming the minimum practical profile shifts for both gears;
(c)* repeat (b) but use the maximum practical profile shifts.
Comment upon the trends suggested by these results.

11. The transmission accuracy level number of a pair of open gears is 6. Further particulars of the 25 mm module 300 mm facewidth gears are as follows :

	number of teeth	allowable stresses, MPa contact	bending	speed, rev/min
pinion	25	1100	290	150
wheel	55	1000	280	-

12. What life may be expected of the gears whilst transmitting 1 MW uniformly? [39 khr]
Shock loading of the foregoing drive results from unsuspected torsional vibration. If the effective application factor is in fact 1.25, what life may now be expected? [5.4 khr]

13.* Two mating gears of commercial quality are to hand with 18 and 56 teeth. Their common facewidth is measured as 50 mm and their addendum diameters as 83.2 and 233.6 mm. Metallurgical analysis reveals that the expected contact and bending stresses of the gears' common material are 1100 and 300 MPa respectively.
Estimate the pair's capacity (kW) for a 10 khr life in a shock-free application in which the pinion speed is 300 rev/min. The transmission accuracy level number is 6. [9.1 kW]

14. A gear pair transmits 75 kW with an application factor of 1.5 and reliability of 99%. Particulars of the commercial 6-accuracy level gears are :

	number of teeth	allowable stresses, MPa contact	bending	speed, rev/min
pinion	20	1300	180	90
wheel	37	1250	175	-

15. Select a suitable module and facewidth for a life of 15 khr. [16, 144 mm]

16. Details of a pair of commercial gears having a transmission accuracy level of 8 are as follows :

	number of teeth	allowable stresses, MPa contact	bending	speed, rev/min
pinion	10	1320	380	200
wheel	36	1100	360	-

17. Select a suitable module and facewidth for a design life of 16 khr whilst transmitting a uniform 125 kW with a reliability of 99%. [16, 187 mm]

18. A commercial gear pair having a transmission accuracy level of 8 is required to transmit 100 kW in a shockfree application with 99% reliability. The speeds of pinion and wheel are 1450 and approximately 470 rev/min. Allowable stresses for contact and for bending of the pinion are 1450 and 400 MPa respectively; for the wheel 1300 and 350 MPa.
    Select suitable tooth numbers and profile shifts, along with a corresponding module and facewidth for a compact pair with a design life of 20 khr.

19. Estimate the life of a gear whose allowable contact stress is 1.2 GPa and which undergoes the stress spectrum :

contact stress	σ_c	(GPa)	1.0	1.1	0.9
speed	N	(rev/min)	500	400	300
duration	t	(hours)	2	1	3

20. [7.9 khr]

21. A pair of 8 mm module, 100 mm facewidth commercial gears is manufactured to a transmission accuracy level of 7 and employed in a periodic duty of 1.25 application factor. The 23 tooth pinion's allowable contact stress is 1.2 GPa at 99% reliability, the 47 tooth wheel's is 1.1 GPa.
    If power is transmitted to the following cycle, what life may be expected of the pair ?

power	P	(kW)	60	45	35
pinion speed	N_1	(rev/min)	200	150	100
duration	t	(min)	10	20	30

22. [13 khr]

Unit3

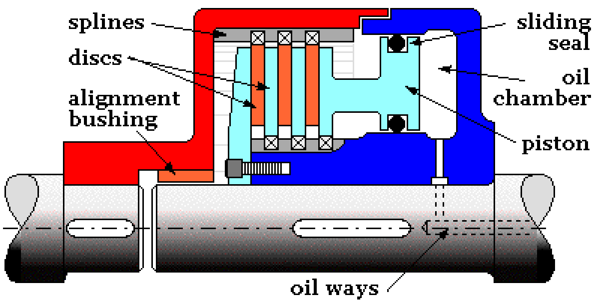
BRAKES

A *brake* decelerates a system by transferring power from it. A *clutch* such as that illustrated (for the most part) accelerates a system by transferring power to it. The two devices in rotary applications are thus very similar as they both transmit torque whilst supporting a varying speed difference across them.

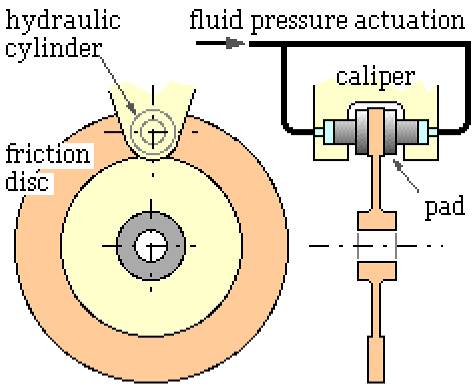
Brakes take a number of forms - for example a system may drive a pump or electric generator, so the pump or generator acts as a brake on the system. However the most

common brakes employ friction to transform the braked system's mechanical energy irreversibly into heat which is then transferred to the surrounding environment - see the flame generated by this sports car's brakes. The friction mechanism is convenient since it allows force and torque to be developed between surfaces which slide over one another due to their different speeds. One of the sliding surfaces is usually metal, the other a special friction material - the *lining* - which is sacrificial. Wear (ie. material loss) of the lining must be catered for, and the lining usually needs to be renewed periodically.

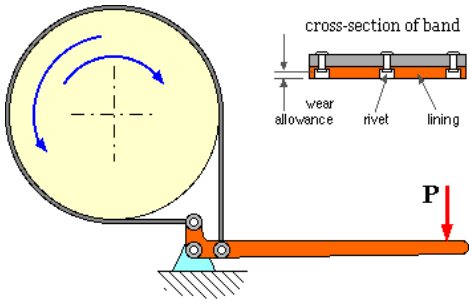
We examine only friction brakes in any detail here - some common embodiements are first described . . . . .



This hydraulically actuated clutch comprises a number of discs faced with lining material which are connected alternately to input and output shafts by torque- transferring splines. The clutch is engaged by high pressure oil applied to an annular piston pressing the discs against one another while they rotate at the different speeds of the two shafts. The normal pressure between discs enable them to exert friction torque on one another which tends to equalise the two speeds.



A hydraulically activated *disc brake* comprises two opposing pistons each faced with a pad of lining material. When the hydraulic pressure is increased the pads are forced against the rotating metal friction disc, exerting a normal force at each contact. The two normal forces cancel one another axially but cause additive tangential friction forces which oppose the disc's motion and decelerate it.

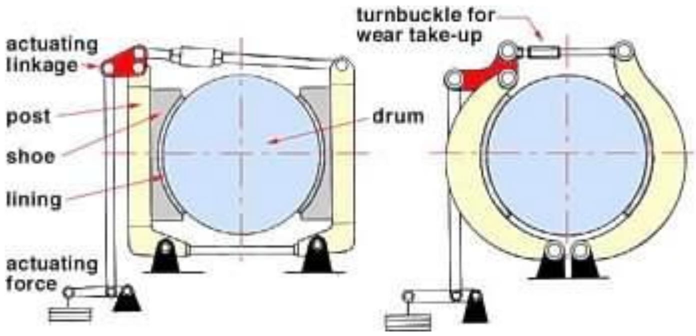


A *band brake* consists of a flexible band faced with friction material bearing on the periphery of a drum which may rotate in either direction.

The *actuation force* P is applied to the band's extremities through an *actuation linkage* such as the cranked lever illustrated. Tension build-up in the band is identical to that in a stationary flat belt.

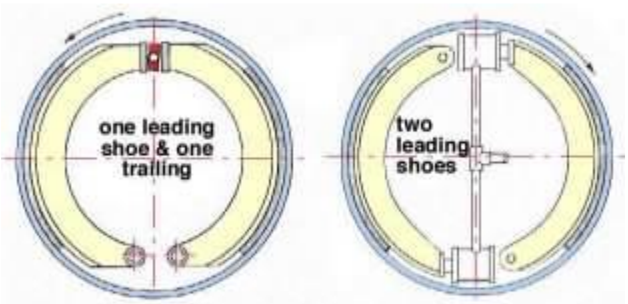
The band cross-section shows lining material riveted to the band. Allowance for lining wear is provided - when the rivets start to rub on the drum they are drilled out prior to new linings being riveted to the band.

The band brake on the left is destined for a fishing trawler winch.



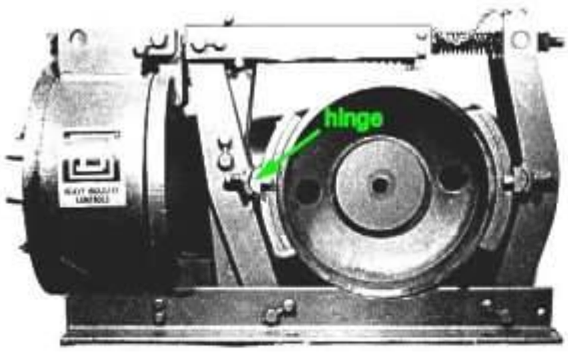
These are *external rigid shoe* brakes - rigid because the shoes with attached linings are rigidly connected to the pivoted posts; external because they lie outside the rotating drum. An actuation linkage distributes the actuation force to the posts thereby causing them both to rotate towards the drum - the linings thus contract around the drum and develop a friction braking torque.

The RH brake features improved hinge locations and integral posts/shoes.



The two hydraulically actuated rigid shoe brakes here are located internal to the drum. The LH brake incorporates a rotating cam which causes the shoes to expand and the linings to bear on the surrounding drum. The RH brake features two leading shoes, enabled by an individual (and more expensive) hydraulic cylinder and piston for each shoe.The terms *leading* and *trailing* are explained below.

These rigid external shoe brakes act on the rope drum of a mine (cage) winder. The arrangement is fail-safe as an electric solenoid disengages the brake to allow motion, but in the event of power failure the brakes are engaged automatically by the large springs visible at the sides of the drum.
Visible in the photograph are the actuation mechanisms with wear-compensating turnbuckles, the electric drive motor on the right, and the cage level indicator on the left.



This is a *hinged shoe* brake - the shoes are hinged to the posts. As wear proceeds the extra degree of freedom allows the linings to conform more closely to the drum than would be the case with rigid shoes. This permits the linings to act more effectively and reduces the need for wear adjustment.
The commercial unit comes complete with actuating solenoid.

About 5% of the heat generated at the sliding interface of a friction brake must be transferred through the lining to the surrounding environment without allowing the lining to reach excessive temperatures, since high temperatures lead to hot spots and distortion, to *fade* (the fall-off in friction coefficient) or, worse, to degradation and charring of the lining which often incoporates organic constituents. Thorough design of a brake therefore requires a detailed transient thermal analysis of the interplay between heat generated by friction, heat transferred through the lining via the surrounding metalwork to the environment, and the instantaneous temperature of the lining surface.

Brake design investigations generally proceed along the following lines :

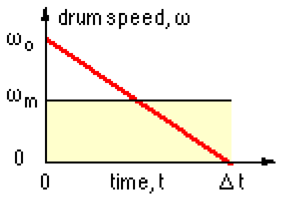
- The braked system is first examined to find out the required brake capacity, that is the torque and average power developed over the braking period.
- The brake is then either selected from a commercially available range or designed

from scratch. In the latter case, conservative rather than optimum brake sizing may be based upon power densities which experience has shown to be acceptable, thus avoiding the difficulties associated with heat transfer appraisal.
- Analysis of the actuating mechanism is necessary to disclose the actuation requirements, brake sensitivity, bearing loads and the like.

The following notes consider these aspects for rotating drum brakes only, and go on to introduce the effect of a road vehicle's braking control system on the vehicle's stability.

System dynamics

The braked system must be analysed to throw light on its braking requirements. Analysis requires knowledge of
- the system's total energy (comprising eg. kinetic, gravitational and elastic potential) initially, ie. before braking
- the system's final total energy ie. after braking
- the initial and final velocities of the brake drum
- the desired braking period Δt, or alternatively the corresponding rotation of the drum $\Delta\theta$



- the life of the brake lining would also be specified or estimated by the designer.

During deceleration, the system is subjected to the essentially constant torque T exerted by the brake, and in the usual situation this constancy implies constant deceleration too. The elementary equations of constant rotational deceleration apply, thus when the brake drum is brought to rest from an initial speed ω_o :-

(1) deceleration $= \omega_o^2 / 2\,\Delta\theta = \omega_o/\Delta t$; $\omega_m = \omega_o/2 = \Delta\theta/\Delta t$
where ω_m is the mean drum speed over the deceleration period.

Application of the work/energy principle to the system enables the torque exerted by the brake and the work done by the brake, U, to be calculated from :-

(2) $U = \Delta E = T\,\Delta\theta$
where ΔE is the loss of system total energy which is absorbed by the brake during deceleration, transformed into heat, and eventually dissipated.

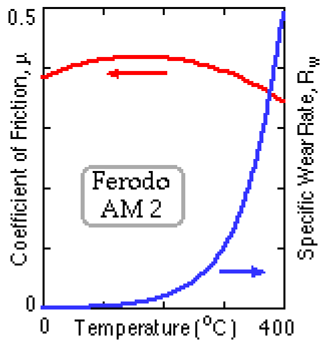
The mean rate of power transformation by the brake over the braking period is :-

(3) $P_m = T\,\omega_m = U / \Delta t$

which forms a basis for the selection or the design of the necessary brake.

Linings

The choice of lining material for a given application is based upon criteria such as the expected coefficient of friction, fade resistance, wear resistance, ease of attachment, rigidity/formability, cost, abrasive tendencies on drum, etc.
Linings traditionally were made from asbestos fibres bound in an organic matrix, however the health risks posed by asbestos have led to the decline of its use.Non-asbestos linings generally consist of three components - metal fibres for strength, modifiers to improve heat conduction, and a phenolic matrix to bind everything together.
The characteristics of Ferodo AM 2, a typical moulded asbestos, are illustrated. The coefficient of friction, which may be taken as 0.39 for design purposes, is not much affected by pressure or by velocity - which should not exceed 18 m/s. The maximum allowable temperature is 400°C.



Linings are attached to shoes either by soft countersunk rivets or by bonding, though set-screws and proprietary fixings may be used in the larger sizes.
In order to withstand the inherent abrasion, mating surfaces should be ferrous with a hardness of at least 150 BHN, or 200 if the duty is heavy. Fine grain high tensile pearlitic iron is generally suitable. The interested reader should refer to manufacturers' publications for further information.

Having ascertained the braking requirements from the system dynamics, we now wish to form some idea of the leading dimensions of a suitable brake.
Practically achievable power density limitations apply to brakes as they do to other mechanical plant such as engines and heat exchangers. For a given size of brake there is a limit to the mechanical power that can be transformed into heat and dissipated without lining temperatures reaching damaging levels. Brake size is characterised by lining contact area, A, so denoting this maximum safe power density as R_p we have, for a reasonable lining life :-

(4) $P_m / A \leq (P_m / A)_{critical} \equiv R_p \quad (kW/m^2)$

Experience suggests the following values of R_p for various types of brake in different applications.

TYPICAL VALUES OF R_p (kW/m^2)

type of duty	*cooling conditions*	*typical applications*	*spot disc brakes*	*drum brakes*	*cone clutches*	*plate brakes & clutches*
Intermittent duty or infrequent full duty applications	Time between applications permits assembly to cool to ambient prior to actuation.	Emergency and safety brakes, safety and torque-limiting clutches.	6000	1800	800	600
Normal intermittent	Some cooling between applications, but temperature builds up to a moderate level over a period of time.	All general duty applications - winding engines, cranes, winches and lifts.	2400	600	400	240
Heavy frequent duty where life is critical.	Frequency of applications too high to permit appreciable cooling between applications.	Presses, drop stamps, excavators and haulage gear.	1200	300	240	120
Typical lining pressure range (kPa)			350-1750	70-700	70-350	70-350

The table indicates that the improved heat transfer capabilities of disc brakes compared to other types enables them to handle greater power densities - per unit area of lining, not necessarily per unit volume of brake. All tabulated power densities should be treated as indicative rather than absolute maxima; their use with (3) and (4) enables reasonable estimates of required lining areas to be made - optimum designs would have to consider thermal analyses, which is beyond the present scope.

An alternative brake rating procedure is based upon the product of average pressure, p_m, over the lining contact area and the mean rubbing speed, v_m, during deceleration. This procedure requires knowledge of the coefficient of friction, μ, so it is less useful than (4) and is mentioned only because it is commonly used - we shall persevere with (4). However if F is the lining contact resultant then :-

(i) $p_m v_m = (F_{normal}/A) v_m = (F_{tangential}/\mu A) v_m = P_m/\mu A = R_p/\mu$
which may be used analogously to (4) to determine the necessary minimum lining area necessary to dissipate a given power if μ is known.

If a drum brake has to be designed for a particular system (rather than chosen from an available range) then the salient brake dimensions may be estimated from the necessary lining area, A, together with a drum diameter- to- lining width ratio somewhere between 3:1 and 10:1, and an angular extent of 100^o say for each of the two shoes.

The lining is sacrificial - it is worn away. The necessary thickness of the lining is therefore dictated by the volume of material lost - this in turn is the product of the total energy dissipated by the lining throughout its life, and the specific wear rate R_w (volume sacrificed per unit energy dissipated) which is a material property and strongly temperature dependent as may be seen from the graph above for Ferodo AM 2. This temperature dependence may be expressed as :-

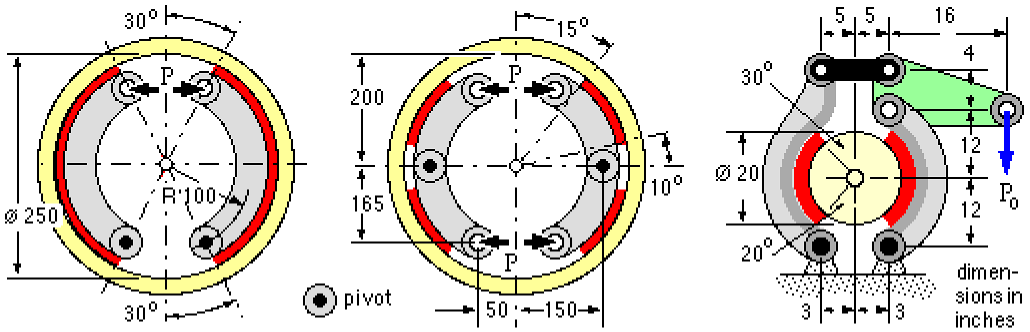
(ii) $R_w \cong R_{wo} \exp ((\text{lining temperature } ^oC / T_o)^n)$
where R_{wo}, T_o and n are constant material properties.

Brake problems

1. A motor whose inertia is 0.3 kg.m^2 drives the rope drum of a hoist through a 5:1 gear reduction. The average diameter, radial thickness and face width of the larger gear's rim are 360, 16 and 50 mm. The mass of the rope drum is 120 kg, its radius of gyration is 110 mm, and it is equipped with grooves of 250 mm pitch diameter, on which is wrapped the hoisting rope whose mass is 0.5 kg/m. The maximum extended length of the rope is 60 m.
   A brake is incorporated into the motor shaft. Determine the brake torque and average power over the braking period when stopping within 1 m, a load of 1 t (1 tonne) being lowered at 3 m/s. [435 Nm, 26.1 kW]

2. Derive equations (**13**).

3. *(Problems 4-6 are similar)*



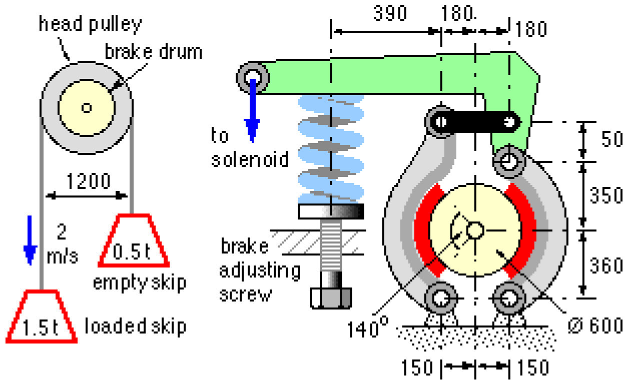
Problem 3 Problem 4 Problems 5,6

4. For each brake, determine the sensitivity, the hinge and drum shaft reactions, and the parameters which are not defined in the duty statements. The brakes are symmetric, except for the mechanism of Problems 5 & 6; the geometry of Problem 6 is identical to that of Problem 5, however the drum rotation is clockwise in 5, counterclockwise in 6.
   Duty statements are *[psi ≡ lbf/in^2; 1000 lbf/kip]* :

problem	3	4	5	6
friction coefficient	0.32	0.24	0.3	0.3
braking torque	-	-	-	25 kip.in
actuation force	1.2 kN	-	400 lbf	-
lining mean pressure limit	-	1.0 MPa	150 psi	-
lining width	28 mm	75 mm	-	1.7 in
Solutions				
braking torque	231 Nm	2646 Nm	2150 lbf.ft	-
actuation force	-	8604 N	-	404 lbf
lining mean pressure	543 kPa	-	-	125 psi
lining width	-	-	1.7 in	-
sensitivity	1.337	1.117	1.143	1.101
hinge reaction - left	462 N	8900 N	6540 lbf	4720 lbf
hinge reaction - right	2716 N	8900 N	4130 lbf	5925 lbf
drum shaft reaction	2323 N	0	2510 lbf	1290 lbf

7. Design an external rigid shoe brake for the hoist of Problem 1, given that braking occurs twice a minute, that the lining is expected to reach a temperature of 300°C, and that the required lining life is 5 khr. A design factor of 1.2 should be applied to the system energy loss of Problem 1.

8. A skip hoist for lifting bulk material, consists of two identical buckets or *"skips"* connected by a wire rope which passes around a motor- driven head pulley. The loaded skip is partially counterbalanced by the empty skip. The head pulley is equipped with the brake illustrated, which is so arranged that in normal operation the brake is disengaged by an electric solenoid. In the event of an electrical power failure however, the brake is automatically engaged by a steel compression spring.



Determine the initial compression of the spring necessary to arrest the loaded skip, falling at 2 m/s, within 2 s of brake engagement.
The inertia of the pulley/drum is 0.5 t.m^2, the coefficient of friction for the brake may be taken as 0.3, and the spring, which is made from 10 mm diameter stock, has 10 active turns of 60 mm mean coil diameter. [27 mm]

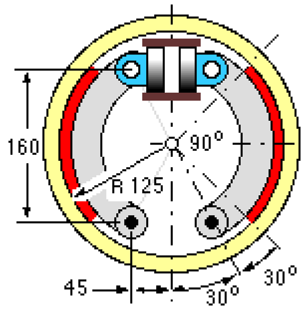
9. At first glance, a sprag type of over-running, uni-directional clutch looks rather like a cylindrical roller bearing in that it consists of two concentric circular rings. However instead of having cylinders between these, there is a series of closely spaced *"sprags"* or cams, similar to the one sketched. These fit loosely for one direction of relative ring rotation to allow *"free-wheeling"*. Light springs keep the sprags in touch with the rings.

A reversal of relative rotation causes a rocking of the sprags and a tightening-up so that a high torque may be carried.
The two lines drawn from the centre O of the rings to the centres of curvature O_1 and O_2 of the sprag surfaces at the contact points, make a small angle α with one another.
Draw the free body of a sprag, perhaps exaggerating the angle α for clarity, and obtain graphically the relative magnitudes and directions of the two contact forces with their normal and tangential components.
Derive approximate equations for the angles betwen the forces and their normal components as functions of the angle α and the ring contact radii r_1 and r_2, and find the maximum value of α in terms of the coefficient of friction μ if slipping is not to occur. [$\alpha_{max} \cong \mu (1 - r_1/r_2)$]

10. The centre of mass of a vehicle lies at $c_F = h = 1/3$
(**a**) Plot the vehicle braking characteristic along with representative loci of constant adhesion coefficient and of constant decelerations;
(**b**) The vehicle is equipped with proportional braking of 1:4 (rear:front). What is the maximum deceleration that can be achieved safely, and the corresponding necessary adhesion coefficient ? [0.4]

(**c**) Repeat (b) if the normalised rear braking force is limited to 0.06. [0.765]

11. The centre of mass of a 1.2 t vehicle lies midway between front and rear axles, at a height above road level of one quarter of the wheelbase (the distance between front and rear axles). The vehicle is equipped with hydraulically operated, symmetric brakes as shown, the front and rear sets being identical except for lining width and hydraulic cylinder diameter.



Further details are :-

	Front	*Rear*
hydraulic cylinder diameter, mm	29	20.5
maximum hydraulic pressure, MPa	5.5	4

12. The friction coefficient between lining and drum is 0.4 and the tyres are 640 mm diameter.
(**a**) Criticise the safety of this braking arrangement.
(**b**) Determine the maximum vehicle deceleration that may be obtained, and the tyre-road adhesion coefficient necessary to achieve it. [7.3 m/s^2, 0.79]
(**c**) Calculate the maximum value of the average lining pressure if the front and rear lining widths are 60 and 40 mm respectively. [1.34 MPa]

13. Repeat Problem 5 with the shoes pivoted to the posts at $b = 11$ in, $\theta_G = 80^o$.
The sensitivity expression for a pivoted shoe is more complex than that of a rigid shoe - the m & n parameters are no longer appropriate. One way to proceed is to denote $' \equiv d/d\mu$, whereupon α' and β' follow from (**6c**) and (**11**). Call the scalar ratio between the pressure components ν and evaluate it from (**11**) as $\nu = N_c / N_s = p_c / p_s = - \beta\, J_s / \beta\, J_c$.
Thereafter form the vector $J_o \equiv J_s + \nu\, J_c$ and hence $\nu' = - \delta\Delta.\beta'\, J_o / \beta\, J_c$. It follows that $\eta = \mu r\, J_o[3] / \alpha J_o$. Differentiating this last expression and inserting into (**vii**) leads to the expression for sensitivity.
[T = 2160 lbf.ft; w = 1.67 in; S = 1.148; R_{HL} = 3610 lbf; R_{HR} = 1400 lbf; R_O = 2475 lbf]