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UNIT I Basics of Mechanisms

- **Introduction:**
  Definitions: Link or Element, Pairing of Elements with degrees of freedom, Grubler’s criterion (without derivation), Kinematic chain, Mechanism, Mobility of Mechanism, Inversions, Machine.

- **Kinematic Chains and Inversions:**
  Kinematic chain with three lower pairs, Four bar chain, Single slider crank chain and Double slider crank chain and their inversions.

- **Mechanisms:**
  1. Quick returns motion mechanisms – Drag link mechanism, Whitworth mechanism and Crank and slotted lever mechanism
  2. Straight line motion mechanisms – Peacelier’s mechanism and Robert’s mechanism.
  4. Toggle mechanism, Pantograph, Hooke’s joint and Ackerman Steering gear mechanism.

**What is Kinematics?**

Kinematics is the study of motion (position, velocity, acceleration). A major goal of understanding kinematics is to develop the ability to design a system that will satisfy specified motion requirements. This will be the emphasis of this class.

**What is Kinetics?**

Kinetics is the study of effect of forces on moving bodies. Good kinematic design should produce good kinetics.

**Definitions**

**Link:**

A link is defined as a member or a combination of members of a mechanism connecting other members and having relative motion between them. The link may consist of one or more resistant bodies. A link may be called as kinematic link or element. Eg: Reciprocating steam engine.
Classification of link is binary, ternary and quarternary.

**Joint:** A connection between two links that allows motion between the links. The motion allowed may be rotational (revolute joint), translational (sliding or prismatic joint), or a combination of the two (roll-slide joint).

**Kinematic pair:**

Kinematic pair is a joint of two links having relative motion between them. The types of kinematic pair are classified according to

- Nature of contact (lower pair, Higher pair)
- Nature of mechanical contact (Closed pair, unclosed pair)
- Nature of relative motion (Sliding pair, turning pair, rolling pair, screw pair, spherical pair)

In the above given Slider crank mechanism, link 2 rotates relative to link 1 and constitutes a revolute or turning pair. Similarly, links 2, 3 and 3, 4 constitute turning pairs. Link 4 (Slider) reciprocates relative to link 1 and its a sliding pair.
Kinematic chain:

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion it is called a kinematic chain.

Eg: The crank shaft of an engine forms a kinematic pair with the bearings which are fixed in a pair, the connecting rod with the crank forms a second kinematic pair, the piston with the connecting rod forms a third pair and the piston with the cylinder forms the fourth pair. The total combination of these links is a kinematic chain. Eg: Lawn mower

![Lawn mower is a combination of kinematic links.](image)

Here, we had to check whether the given link is a kinematic chain

We can use two formulas

1. \( l = 2p - 4 \)
2. \( j = (3/2)l - 2 \)

Exercise: Check 3 links, 4 links, 5 link arrangements is a kinematic chain.
Mechanism

If motion of any of the movable links results in definite motions of the others the linkage is known as mechanism.

Machine

When a mechanism is required to transmit power or to do some particular type of work it then becomes a machine.

Degrees of Freedom

It is defined as the number of input parameters which must be independently controlled in order to bring the mechanism in to useful engineering purposes.

It is also defined as the number of independent relative motions, both translational and rotational, a pair can have.

Degrees of freedom = 6 – no. of restraints.

To find the number of degrees of freedom for a plane mechanism we have Grubler’s equation

\[ F = 3(n - 1) - 2j_1 - j_2 \]

F = Mobility or number of degrees of freedom
n = Number of links including frame.
\( j_1 \) = Joints with single (one) degree of freedom.
\( J_2 \) = Joints with two degrees of freedom.
F > 0, results in a mechanism with ‘F’ degrees of freedom.
F = 0, results in a statically determinate structure.
F < 0, results in a statically indeterminate structure.

MOBILITY- Kutzbach criterion, Grashoff's law

Mobility: Kutzbach criterion:

Fundamental Equation for 2-D Mechanisms: \[ M = 3(L - 1) - 2J_1 - J_2 \]

Here we solve, the following problems
• If you pin one end of the link to the plane, how many degrees of freedom does it now have?
• Add a second link to the picture so that you have one link pinned to the plane and one free to move in the plane. How many degrees of freedom exist between the two links? (4 is the correct answer)
• Pin the second link to the free end of the first link. How many degrees of freedom do you now have?
• How many degrees of freedom do you have each time you introduce a moving link? How many degrees of freedom do you take away when you add a simple joint? How many degrees of freedom would you take away by adding a half joint? Do the different terms in equation make sense in light of this knowledge?

Here we would also calculate the following:

1. Number of binary links
2. Number of ternary links
3. The number of quaternary links
4. number of loops
5. number of joints or pairs
6. the number of degrees of freedom
7. Stating whether the given linkages are mechanisms with one degree of freedom.

**Grashoff's law:**

- **Grashoff 4-bar linkage:**
  A linkage that contains one or more links capable of undergoing a full rotation.

  A linkage is Grashoff if:  \( S + L < P + Q \)
  (Where: \( S \) = shortest link length, \( L \) = longest, \( P, Q \) = intermediate length links).

  Both joints of the shortest link are capable of 360 degrees of rotation in a Grashoff linkages.

  This gives us 4 possible linkages:

  • crank-rocker (input rotates 360)
  • rocker-crank-rocker (coupler rotates 360)
  • rocker-crank (follower)
  • double crank (all links rotate 360).
Problems:

- We can also calculate how many unique mechanisms can be obtained from the 8-link kinematic chain.

- We can also calculate degree of freedom and the number of ternary and quaternary links it will have if it has only single turning pairs.

Inversions of four bar chain mechanism:
There are three inversions: 1) Beam Engine or Crank and lever mechanism. 2) Coupling rod of locomotive or double crank mechanism. 3) Watt’s straight line mechanism or double lever mechanism.

Beam Engine:
When the crank AB rotates about A, the link CE pivoted at D makes vertical reciprocating motion at end E. This is used to convert rotary motion to reciprocating motion and vice versa. It is also known as Crank and lever mechanism. This mechanism is shown in the figure below.

Coupling rod of locomotive: In this mechanism the length of link AD = length of link C. Also length of link AB = length of link CD. When AB rotates about A, the crank DC rotates about D. This mechanism is used for coupling locomotive wheels. Since links AB and CD work as cranks, this mechanism is also known as double crank mechanism. This is shown in the figure below.

Watt’s straight line mechanism or Double lever mechanism: In this mechanism, the links AB & DE act as levers at the ends A & E of these levers are fixed. The AB & DE are parallel in the mean position of the mechanism and coupling rod BD is perpendicular to the levers AB & DE. On any small displacement of the mechanism the tracing point ‘C’ traces the shape of number
Inversions of Slider crank Chain:
It is a four bar chain having one sliding pair and three turning pairs. It is shown in the figure below the purpose of this mechanism is to convert rotary motion to reciprocating motion and vice versa.

There are four inversions in a single slider chain mechanism. They are:

1) Reciprocating engine mechanism (1st inversion)
2) Oscillating cylinder engine mechanism (2nd inversion)
3) Crank and slotted lever mechanism (2nd inversion)
4) Whitworth quick return motion mechanism (3rd inversion)
5) Rotary engine mechanism (3rd inversion)
6) Bull engine mechanism (4th inversion)
7) Hand Pump (4th inversion)

Reciprocating engine mechanism:
In the first inversion, the link 1 i.e., the cylinder and the frame is kept fixed. The fig below shows a reciprocating engine.
A slotted link 1 is fixed. When the crank 2 rotates about O, the sliding piston 4 reciprocates in the slotted link 1. This mechanism is used in steam engine, pumps, compressors, I.C. engines, etc.

**Crank and slotted lever mechanism:**
It is an application of second inversion. The crank and slotted lever mechanism is shown in figure below.

In this mechanism link 3 is fixed. The slider (link 1) reciprocates in oscillating slotted lever (link 4) and crank (link 2) rotates. Link 5 connects link 4 to the ram (link 6). The ram with the cutting tool reciprocates perpendicular to the fixed link 3. The ram with the tool reverses its direction of motion when link 2 is perpendicular to link 4. Thus the cutting stroke is executed during the rotation of the crank through angle α and the return stroke is executed when the crank rotates through angle β or $360^\circ - \alpha$. Therefore, when the crank rotates uniformly, we get,

\[
\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \quad \text{or} \quad \frac{360^\circ - \alpha}{\alpha}
\]

This mechanism is used in shaping machines, slotting machines and in rotary engines.
Whitworth quick return motion mechanism:

Third inversion is obtained by fixing the crank i.e. link 2. Whitworth quick return mechanism is an application of third inversion. This mechanism is shown in the figure below. The crank OC is fixed and OQ rotates about O. The slider slides in the slotted link and generates a circle of radius CP. Link 5 connects the extension OQ provided on the opposite side of the link 1 to the ram (link 6). The rotary motion of P is taken to the ram R which reciprocates. The quick return motion mechanism is used in shapers and slotting machines. The angle covered during cutting stroke from P1 to P2 in counter clockwise direction is \( \alpha \) or \( 360^\circ - 2\theta \). During the return stroke, the angle covered is \( 2\theta \) or \( \beta \).

Rotary engine mechanism or Gnome Engine:

Rotary engine mechanism or gnome engine is another application of third inversion. It is a rotary cylinder \( V \) – type internal combustion engine used as an aero – engine. But now Gnome engine has been replaced by Gas turbines. The Gnome engine has generally seven cylinders in one plane. The crank OA is fixed and all the connecting rods from the pistons are connected to A. In
this mechanism when the pistons reciprocate in the cylinders, the whole assembly of cylinders, pistons and connecting rods rotate about the axis O, where the entire mechanical power developed, is obtained in the form of rotation of the crank shaft. This mechanism is shown in the figure below.

Double Slider Crank Chain:
A four bar chain having two turning and two sliding pairs such that two pairs of the same kind are adjacent is known as double slider crank chain.

Inversions of Double slider Crank chain:
It consists of two sliding pairs and two turning pairs. There are three important inversions of double slider crank chain. 1) Elliptical trammel. 2) Scotch yoke mechanism. 3) Oldham’s Coupling.

Elliptical Trammel:
This is an instrument for drawing ellipses. Here the slotted link is fixed. The sliding block P and Q in vertical and horizontal slots respectively. The end R generates an ellipse with the displacement of sliders P and Q.
Let us take $OX$ and $OY$ as horizontal and vertical axes and let the link $BA$ is inclined at an angle $\theta$ with the horizontal, as shown in Fig. 534 (b). Now the co-ordinates of the point $P$ on the link $BA$ will be
\[ x = PQ = AP \cos \theta; \quad \text{and} \quad y = PR = BP \sin \theta \]
or
\[ \frac{x}{AP} = \cos \theta; \quad \text{and} \quad \frac{y}{BP} = \sin \theta \]

Squaring and adding,
\[ \frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2 \theta + \sin^2 \theta = 1 \]

This is the equation of an ellipse. Hence the path traced by point $P$ is an ellipse whose semi-major axis is $AP$ and semi-minor axis is $BP$.

**Note:** If $P$ is the mid-point of link $BA$, then $AP = BP$. The above equation can be written as
\[ \frac{x^2}{(AP)^2} + \frac{y^2}{(AP)^2} = 1 \quad \text{or} \quad x^2 + y^2 = (AP)^2 \]

This is the equation of a circle whose radius is $AP$. Hence if $P$ is the mid-point of link $BA$, it will trace a circle.

**Scotch yoke mechanism:** This mechanism is used to convert rotary motion in to reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. Link 1 is fixed. In this mechanism when the link 2 rotates about $B$ as centre, the link 4 reciprocates. The fixed link 1 guides the frame.

![Scotch yoke mechanism](image)

**Oldham’s coupling:** The third inversion of obtained by fixin the link connecting the 2 blocks P & Q. If one block is turning through an angle, the frame and the other block will also turn through the same angle. It is shown in the figure below.
An application of the third inversion of the double slider crank mechanism is Oldham’s coupling shown in the figure. This coupling is used for connecting two parallel shafts when the distance between the shafts is small. The two shafts to be connected have flanges at their ends, secured by forging. Slots are cut in the flanges. These flanges form 1 and 3. An intermediate disc having tongues at right angles and opposite sides is fitted in between the flanges. The intermediate piece forms the link 4 which slides or reciprocates in flanges 1 & 3. The link two is fixed as shown. When flange 1 turns, the intermediate disc 4 must turn through the same angle and whatever angle 4 turns, the flange 3 must turn through the same angle. Hence 1, 4 & 3 must have the same angular velocity at every instant. If the distance between the axis of the shaft is $x$, it will be the diameter if the circle traced by the centre of the intermediate piece. The maximum sliding speed of each tongue along its slot is given by $v=x\omega$ where, $\omega =$ angular velocity of each shaft in rad/sec $v =$ linear velocity in m/sec
EXERCISES

1. In a crank and slotted lever quick return mechanism, the distance between the fixed centres is 150 mm and the driving crank is 75 mm long. Determine the ratio of the time taken on the cutting and return strokes.  
   [Ans. 2]

2. In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres $O$ and $C$ is 200 mm. The driving crank $CP$ is 75 mm long. The pin $Q$ on the slotted lever, 360 mm from the fulcrum $O$, is connected by a link $QR$ 100 mm long, to a pin $R$ on the ram. The line of stroke of $R$ is perpendicular to $OC$ and intersects $OC$ produced at a point 150 mm from $C$. Determine the ratio of times taken on the cutting and return strokes.  
   [Ans. 1.647]

3. In a crank and slotted lever quick return mechanism, as shown in Fig. 5.37, the driving crank length is 75 mm. The distance between the fixed centres is 200 mm and the length of the slotted lever is 500 mm. Find the ratio of the times taken on the cutting and idle strokes. Determine the effective stroke also.  
   [Ans. 1.67 ; 380 mm]

Mechanical Advantage, Transmission angle:
- The mechanical advantage (MA) is defined as the ratio of output torque to the input torque. (Or) ratio of load to output.
- Transmission angle.
- The extreme values of the transmission angle occur when the crank lies along the line of frame.
common mechanisms-Single, Double and offset slider mechanisms - Quick return mechanisms:

Quick Return Motion Mechanisms:
Many times mechanisms are designed to perform repetitive operations. During these operations for a certain period the mechanisms will be under load known as working stroke and the remaining period is known as the return stroke, the mechanism returns to repeat the operation without load. The ratio of time of working stroke to that of the return stroke is known a time ratio. Quick return mechanisms are used in machine tools to give a slow cutting stroke and a quick return stroke. The various quick return mechanisms commonly used are i) Whitworth ii) Drag link. iii) Crank and slotted lever mechanism.

Whitworth quick return mechanism:
Whitworth quick return mechanism is an application of third inversion of the single slider crank chain. This mechanism is shown in the figure below. The crank OC is fixed and OQ rotates about O. The slider slides in the slotted link and generates a circle of radius CP. Link 5 connects the extension OQ provided on the opposite side of the link 1 to the ram (link 6). The rotary motion of P is taken to the ram R which reciprocates. The quick return motion mechanism is used in shapers and slotting machines.

![Whitworth quick return mechanism diagram](image)

The angle covered during cutting stroke from P1 to P2 in counter clockwise direction is α or 360 -2θ. During the return stroke, the angle covered is 2θ or β.

Drag link mechanism:

![Drag link mechanism diagram](image)

This is four bar mechanism with double crank in which the shortest link is fixed. If the crank AB
rotates at a uniform speed, the crank CD rotate at a non-uniform speed. This rotation of link CD is transformed to quick return reciprocity motion of the ram E by the link CE as shown in figure. When the crank AB rotates through an angle $\alpha$ in Counter clockwise direction during working stroke, the link CD rotates through 180. We can observe that $\frac{\alpha}{\beta} >$. Hence time of working stroke is $\frac{\alpha}{\beta}$ times more or the return stroke is $\frac{\alpha}{\beta}$ times quicker. Shortest link is always stationary link. Sum of the shortest and the longest links of the four links 1, 2, 3 and 4 are less than the sum of the other two. It is the necessary condition for the drag link quick return mechanism.

**Crank and slotted lever mechanism:**
It is an application of second inversion. The crank and slotted lever mechanism is shown in figure below.

In this mechanism link 3 is fixed. The slider (link 1) reciprocates in oscillating slotted lever (link 4) and crank (link 2) rotates. Link 5 connects link 4 to the ram (link 6). The ram with the cutting tool reciprocates perpendicular to the fixed link 3. The ram with the tool reverses its direction of motion when link 2 is perpendicular to link 4. Thus the cutting stroke is executed during the rotation of the crank through angle $\alpha$ and the return stroke is executed when the crank rotates through angle $\beta$ or $360 - \alpha$. Therefore, when the crank rotates uniformly, we get,

\[
\text{Time to cutting} = \alpha = \frac{\alpha}{\beta} \\
\text{Time of return} = \beta = 360 - \alpha
\]

This mechanism is used in shaping machines, slotting machines and in rotary engines.

**Ratchets and escapements - Indexing Mechanisms - Rocking Mechanisms:**

**Intermittent motion mechanism:**

**Ratchet and Pawl mechanism:** This mechanism is used in producing intermittent rotary motion member. A ratchet and Pawl mechanism consists of a ratchet wheel 2 and a pawl 3 as shown in the figure. When the lever 4 carrying pawl is raised, the ratchet wheel rotates in the counter clock wise direction (driven by pawl). As the pawl lever is lowered the pawl slides over the ratchet teeth. One more pawl 5 is used to prevent the ratchet from reversing. Ratchets are used in feed mechanisms, lifting jacks, clocks, watches and counting devices.
**Geneva mechanism:** Geneva mechanism is an intermittent motion mechanism. It consists of a driving wheel D carrying a pin P which engages in a slot of follower F as shown in figure. During one quarter revolution of the driving plate, the Pin and follower remain in contact and hence the follower is turned by one quarter of a turn. During the remaining time of one revolution of the driver, the follower remains in rest locked in position by the circular arc.

**Pantograph:** Pantograph is used to copy the curves in reduced or enlarged scales. Hence this mechanism finds its use in copying devices such as engraving or profiling machines.
This is a simple figure of a Pantograph. The links are pin jointed at A, B, C and D. AB is parallel to DC and AD is parallel to BC. Link BA is extended to fixed pin O. Q is a point on the link AD. If the motion of Q is to be enlarged then the link BC is extended to P such that O, Q and P are in a straight line. Then it can be shown that the points P and Q always move parallel and similar to each other over any path straight or curved. Their motions will be proportional to their distance from the fixed point. Let ABCD be the initial position. Suppose if point Q moves to Q1, then all the links and the joints will move to the new positions (such as A moves to A1, B moves to Q1, C moves to Q1, D moves to D1 and P to P1) and the new configuration of the mechanism is shown by dotted lines. The movement of Q (Q Q1) will be enlarged to PP1 in a definite ratio.

**Toggle Mechanism:**

![Toggle Mechanism Diagram](image)

In slider crank mechanism as the crank approaches one of its dead centre position, the slider approaches zero. The ratio of the crank movement to the slider movement approaching infinity is proportional to the mechanical advantage. This is the principle used in toggle mechanism. A toggle mechanism is used when large forces act through a short distance is required. The figure below shows a toggle mechanism. Links CD and CE are of same length. Resolving the forces at C vertically $F \sin \alpha = P \cos \alpha$  

Therefore, $F = P$. (because $\sin \alpha/\cos \alpha = \tan \alpha$) $2 \tan \alpha$ Thus for the given value of P, as the links CD and CE approaches collinear position ($\alpha O$), the force F rises rapidly.

**Hooke’s joint:**

Hooke’s joint used to connect two parallel intersecting shafts as shown in figure. This can also be used for shaft with angular misalignment where flexible coupling does not serve the purpose. Hence Hooke’s joint is a means of connecting two rotating shafts whose axes lie in the same direction.
plane and their directions making a small angle with each other. It is commonly known as Universal joint. In Europe it is called as Cardan joint.

5. **Ackermann steering gear mechanism:**

![Ackermann steering gear mechanism diagram]

This mechanism is made of only turning pairs and is made of only turning pairs wear and tear of the parts is less and cheaper in manufacturing. The cross link KL connects two short axles AC and BD of the front wheels through the short links AK and BL which forms bell crank levers CAK and DBL respectively as shown in fig, the longer links AB and KL are parallel and the shorter links AK and BL are inclined at an angle $\alpha$. When the vehicles steer to the right as shown in the figure, the short link BL is turned so as to increase $\alpha$, whereas the link LK causes the other short link AK to turn so as to reduce $\alpha$. The fundamental equation for correct steering is, $\cot\Phi - \cos\theta = \frac{b}{l}$. In the above arrangement it is clear that the angle $\Phi$ through which AK turns is less than the angle $\theta$ through which the BL turns and therefore the left front axle turns through a smaller angle than the right front axle. For different angle of turn $\theta$, the corresponding value of $\Phi$ and $(\cot\Phi - \cos\theta)$ are noted. This is done by actually drawing the mechanism to a scale or by calculations. Therefore for different value of the corresponding value of and are tabulated. Approximate value of $b/l$ for correct steering should be between 0.4 and 0.5. In an Ackermann steering gear mechanism, the instantaneous centre I does not lie on the axis of the rear axle but on a line parallel to the rear axle axis at an approximate distance of 0.31 above it.

**Question Bank- I unit**

**Important Two marks**

- Define resistant body.
- Define Link or Element
- Differentiate Machine and Structure
- Define Kinematic Pair.
- Define Kinematic Chain.
- What are the types of joints?
- Define Degrees of Freedom (Mobility).
- Write down the Kutzbach criterion for plane mechanism.
- What is meant by spatial mechanism?
- Differentiate Machine and Mechanism.
- Define Inversion of Mechanism
• Define Grashof’s Law.
• Define Pantograph.
• What is Elliptical Trammel?
• Define Transmission Angle and Mechanical Advantage
• What is a straight line Mechanism?

**Important Descriptive University questions:**

• Explain different types of Link.
• Classify and explain the Kinematic pair.
• Explain any two inversion of four bar chain.
• Explain the first inversion of Single Slider Crank Chain.
• Explain first inversion of Double Slider crank chain.
• Explain third inversion of double slider crank chain.
• Explain the offset slider crank mechanism.
• Explain Straight line mechanism with neat sketch.
• With the help of a neat sketch explain the working of Oldham’s coupling.
• Explain steering gear mechanism with neat sketch.

• With the help of a neat sketch explain the working of Whitworth quick return mechanism.

• With the help of a neat sketch explain the working of Single slider and double slider crank chain mechanism.

**References**


**Web site:**

www.nptel.iitm.ac.in (National programme on technology enhanced learning by IIT, India)

### Unit II  KINEMATICS ANALYSIS

#### Graphical method

Analysis of simple mechanisms- Slider crank mechanism and four bar mechanism- for displacement, velocity and acceleration. Shaping machine mechanism- coincident points- coriolis acceleration

#### Analytical method

Analysis of simple mechanisms- Slider crank mechanism and four bar mechanism- Approximate analytical expression for displacement, velocity and acceleration of piston of reciprocating engine mechanism

**Velocity and Acceleration analysis of mechanisms:**

Velocity and acceleration analysis by vector polygons: Relative velocity and accelerations of particles in a common link, relative velocity and accelerations of coincident particles on separate link, Coriolis component of acceleration.

Velocity and acceleration analysis by complex numbers: Analysis of single slider crank mechanism and four bar mechanism by loop closure equations and complex numbers.

**Displacement, velocity and acceleration analysis in simple mechanisms:**

**Important Concepts in Velocity Analysis**

1. The absolute velocity of any point on a mechanism is the velocity of that point with reference to ground.

2. Relative velocity describes how one point on a mechanism moves relative to another point on the mechanism.

3. The velocity of a point on a moving link relative to the pivot of the link is given by the equation: \( V = \omega r \), where \( \omega \) = angular velocity of the link and \( r \) = distance from pivot.

**Acceleration Components**

- **Normal Acceleration:** \( A^n = \omega^2 r \). Points toward the center of rotation
- **Tangential Acceleration:** \( A^t = \dot{r} \). In a direction perpendicular to the link
- **Coriolis Acceleration:** \( A^c = 2\omega \frac{dr}{dt} \). In a direction perpendicular to the link
- **Sliding Acceleration:** \( A^s = \frac{d^2r}{dt^2} \). In the direction of sliding.

A rotating link will produce normal and tangential acceleration components at any point a distance, \( r \), from the rotational pivot of the link. The total acceleration of that point is the vector sum of the components.
A slider attached to ground experiences only sliding acceleration.

A slider attached to a rotating link (such that the slider is moving in or out along the link as the link rotates) experiences all 4 components of acceleration. Perhaps the most confusing of these is the coriolis acceleration, though the concept of coriolis acceleration is fairly simple. Imagine yourself standing at the center of a merry-go-round as it spins at a constant speed ($\omega$). You begin to walk toward the outer edge of the merry-go-round at a constant speed ($dr/dt$). Even though you are walking at a constant speed and the merry-go-round is spinning at a constant speed, your total velocity is increasing because you are moving away from the center of rotation (i.e. the edge of the merry-go-round is moving faster than the center). This is the coriolis acceleration. In what direction did your speed increase? This is the direction of the coriolis acceleration.

The total acceleration of a point is the vector sum of all applicable acceleration components:

$$\mathbf{A} = \mathbf{A}^n + \mathbf{A}^i + \mathbf{A}^c + \mathbf{A}^s$$

These vectors and the above equation can be broken into x and y components by applying sines and cosines to the vector diagrams to determine the x and y components of each vector. In this way, the x and y components of the total acceleration can be found.

**Graphical Method, Velocity and Acceleration polygons:**

**Graphical velocity analysis:**

It is a very short step (using basic trigonometry with sines and cosines) to convert the graphical results into numerical results. The basic steps are these:

1. Set up a velocity reference plane with a point of zero velocity designated.
2. Use the equation, $V = \omega r$, to calculate any known linkage velocities.
3. Plot your known linkage velocities on the velocity plot. A linkage that is rotating about ground gives an absolute velocity. This is a vector that originates at the zero velocity point and runs perpendicular to the link to show the direction of motion. The vector, $\mathbf{v}_A$, gives the velocity of point A.
4. Plot all other velocity vector directions. A point on a grounded link (such as point B) will produce an absolute velocity vector passing through the zero velocity point and perpendicular to the link. A point on a floating link (such as B relative to point A) will produce a relative velocity vector. This vector will be perpendicular to the link AB and pass through the reference point (A) on the velocity diagram.

5. One should be able to form a closed triangle (for a 4-bar) that shows the vector equation: \( V_B = V_A + V_{B/A} \) where \( V_B \) = absolute velocity of point B, \( V_A \) = absolute velocity of point A, and \( V_{B/A} \) is the velocity of point B relative to point A.

**Velocity Analysis of Four Bar Mechanisms:**
- Problems solving in Four Bar Mechanisms and additional links.

**Velocity Analysis of Slider Crank Mechanisms:**
- Problems solving in Slider Crank Mechanisms and additional links.

**Acceleration Analysis of Four Bar Mechanisms:**
- Problems solving in Four Bar Mechanisms and additional links.

**Acceleration Analysis of Slider Crank Mechanisms:**
- Problems solving in Slider Crank Mechanisms and additional links.

**Kinematic analysis by Complex Algebra methods:**
- Analysis of single slider crank mechanism and four bar mechanism by loop closure equations and complex numbers.

**vector Approach:**
- Relative velocity and accelerations of particles in a common link, relative velocity and accelerations of coincident particles on separate link

**Computer applications in the kinematic analysis of simple mechanisms:**
- Computer programming for simple mechanisms

**Coincident points, Coriolis Acceleration:**
- **Coriolis Acceleration:** \( A^c = 2\omega \frac{dr}{dt} \). In a direction perpendicular to the link. A slider attached to ground experiences only sliding acceleration. A slider attached to a rotating link (such that the slider is moving in or out along the link as the link rotates) experiences all 4 components of acceleration. Perhaps the most confusing of these is the coriolis acceleration, though the concept of coriolis acceleration is fairly simple. Imagine yourself standing at the center of a merry-go-round as it spins at a constant speed (\( \omega \)). You begin to walk toward the outer edge of the merry-go-round at a constant speed (\( \frac{dr}{dt} \)). Even though you are walking at a constant speed and the merry-go-round is spinning at a constant speed, your total velocity is increasing because you are moving away from the center of rotation (i.e. the edge of the merry-go-round is moving faster than the center). This is the coriolis acceleration. In what direction did your speed increase? This is the direction of the coriolis acceleration.
1. The direction of linear velocity of any point on a link with respect to another point on the same link is
   (a) parallel to the link joining the points
   (b) perpendicular to the link joining the points
   (c) at 45° to the link joining the points
   (d) none of these
2. The magnitude of linear velocity of a point B on a link AB relative to point A is
   (a) \( \omega AB \)
   (b) \( \omega (AB)^2 \)
   (c) \( \omega^2 . AB \)
   (d) \( (\omega . AB)^2 \)
   Where \( \omega \) = Angular velocity of the link AB.
3. The two links OA and OB are connected by a pin joint at O. If the link OA turns with angular velocity \( \omega_1 \) rad/s in the clockwise direction and the link OB turns with angular velocity \( \omega_2 \) rad/s in the anti-clockwise direction, then the rubbing velocity at the pin joint O is
   (a) \( \omega_1 \omega_2 r \)
   (b) \( (\omega_1 - \omega_2) r \)
   (c) \( (\omega_1 + \omega_2) r \)
   (d) \( (\omega_1 - \omega_2) 2 r \)
   Where \( r \) = Radius of the pin at O.
4. In the above question, if both the links OA and OB turn in clockwise direction, then the rubbing velocity at the pin joint O is
   (a) \( \omega_1 \omega_2 r \)
   (b) \( (\omega_1 - \omega_2) r \)
   (c) \( (\omega_1 + \omega_2) r \)
   (d) \( (\omega_1 - \omega_2) 2 r \)
5. In a four bar mechanism, as shown in Fig. 7.43, if a force \( F_A \) is acting at point A in the direction of its velocity \( v_A \), and a force \( F_B \) is transmitted to the joint B in the direction of its velocity \( v_B \), then the ideal mechanical advantage is equal to
   (a) \( F_B v_A \)
   (b) \( F_A v_B \)
   (c) \( \frac{F_B}{v_B} \)
   (d) \( \frac{F_B}{F_A} \)

**ANSWERS**

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**Question bank – II unit**

**Important two questions**

1. What are the types of motion?
2. What is configuration diagram or space diagram?
3. Define Coincident points
4. Define Coriolis Component
5. Write down the expression for Coriolis component of acceleration defining each of terms in the expression.
6. What are the types of instantaneous Centres?
7. Define Kennedy’s theorem.
8. Define rubbing velocity at a pin joint.
9. What are the various methods used for finding out velocity of mechanism?
10. Define Instantaneous centre.
11. Define Angular Velocity ratio theorem.

**Important descriptive university questions:**

- The Crank of a slider crank mechanisms rotates clockwise at a Constant speed of 600 r.p.m. The crank is 125 mm and connecting rod is 500 mm long. Determine 1. Linear velocity and acceleration of the mid Point of the connecting rod, and 2. Angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.
- In a four link mechanism, the dimensions of the links are AB=200 mm, BC=400 mm, CD=450 mm and AD=600 mm. At the instant when DAB=90°, the link AB has angular velocity of 36 rad/s in the clockwise direction. Determine (i) The velocity of point C, (ii) The velocity of point E on the link BC When BE =200 mm (iii) the angular velocities of links BC and CD, iv) acceleration of link of link BC.
- Derive the expressions for Velocity and acceleration of piston in reciprocating steam engine mechanism with neat sketch
- Derive the expression for Coriolis component of acceleration with neat sketch
- In a slider crank mechanism, the length of the crank and the connecting rod are 100 mm and 400 mm respectively./ The crank [position is 45° from IDC, the crank shaft speed is 600 r.p.m., Clockwise. Using analytical method Determine (1) Velocity and acceleration of the slider, and (2) Angular velocity and angular acceleration of the connecting rod.
- Locate all instantaneous centers of the slider crank mechanism; the length of crank OB and Connecting rod AB are 125 mm and 500 mm respectively. The crank speed is 600 rpm clockwise. When the crank has turned 45° from the IDC, Determine (i)velocity of slider’ A’ (ii)Angular Velocity of connecting rod ‘AB’.

**References**


**Web site:**

- www. nptel.iitm.ac.in (National programme on technology enhanced learning  by IIT, India)
3. KINEMATICS OF CAMS

INTRODUCTION

A cam is a mechanical device used to transmit motion to a follower by direct contact. The driver is called the cam and the driven member is called the follower. In a cam follower pair, the cam normally rotates while the follower may translate or oscillate. A familiar example is the camshaft of an automobile engine, where the cams drive the push rods (the followers) to open and close the valves in synchronization with the motion of the pistons.

Types of cams

Cams can be classified based on their physical shape.

a) Disk or plate cam: The disk (or plate) cam has an irregular contour to impart a specific motion to the follower. The follower moves in a plane perpendicular to the axis of rotation of the camshaft and is held in contact with the cam by springs or gravity.

Fig. 3.1 Plate or disk cam.

b) Cylindrical cam: The cylindrical cam has a groove cut along its cylindrical surface. The roller follows the groove, and the follower moves in a plane parallel to the axis of rotation of the cylinder.

Fig. 3.2 Cylindrical cam.
c) Translating cam. The translating cam is a contoured or grooved plate sliding on a guiding surface(s). The follower may oscillate (Fig. 3.3a) or reciprocate (Fig. 3.3b). The contour or the shape of the groove is determined by the specified motion of the follower.

Fig. 3.3 translating cam

Types of followers(Fig 3.4):

(i) Based on surface in contact.

(a) Knife edge follower

(b) Roller follower

(c) Flat faced follower
(d) Spherical follower

Fig 3.4

(ii) Based on type of motion (Fig 3.5):

(a) Oscillating follower

(b) Translating follower

Fig: 3.5

Based on line of motion (Fig 3.6):
(a) Radial follower: The lines of movement of in-line cam followers pass through the centers of the camshafts

(b) Off-set follower: For this type, the lines of movement are offset from the centers of the camshafts

**nomenclature (Fig. 3.7):**

*Cam Profile* The contour of the working surface of the cam.

*Tracer Point* The point at the knife edge of a follower, or the center of a roller, or the center of a spherical face.

*Pitch Curve* The path of the tracer point.

*Base Circle* The smallest circle drawn, tangential to the cam profile, with its center on the axis of the camshaft. The size of the base circle determines the size of the cam.

*Prime Circle* The smallest circle drawn, tangential to the pitch curve, with its center on the axis of the camshaft.

*Pressure Angle* The angle between the normal to the pitch curve and the direction of motion of the follower at the point of contact.
Fig 3.7

Types of follower motion:

Cam follower systems are designed to achieve a desired oscillatory motion. Appropriate displacement patterns are to be selected for this purpose, before designing the cam surface. The cam is assumed to rotate at a constant speed and the follower raises, dwells, returns to its original position and dwells again through specified angles of rotation of the cam, during each revolution of the cam. Some of the standard follower motions are as follows:

They are, follower motion with,
(a) Uniform velocity

(b) Modified uniform velocity

(c) Uniform acceleration and deceleration

(d) Simple harmonic motion

(e) Cycloidal motion

**Displacement diagrams:** In a cam follower system, the motion of the follower is very important. Its displacement can be plotted against the angular displacement \( \theta \) of the cam and it is called as the displacement diagram. The displacement of the follower is plotted along the y-axis and angular displacement \( \theta \) of the cam is plotted along x-axis. From the displacement diagram, velocity and acceleration of the follower can also be plotted for different angular displacements \( \theta \) of the cam. The displacement, velocity and acceleration diagrams are plotted for one cycle of operation i.e., one rotation of the cam. Displacement diagrams are basic requirements for the construction of cam profiles. Construction of displacement diagrams and calculation of velocities and accelerations of followers with different types of motions are discussed in the following sections.

(a) **Follower motion with Uniform velocity:**

Fig.3.8 shows the displacement, velocity and acceleration patterns of a follower having uniform velocity type of motion. Since the follower moves with constant velocity, during rise and fall, the displacement varies linearly with \( \theta \). Also, since the velocity changes from zero to a finite value, within no time, theoretically, the acceleration becomes infinite at the beginning and end of rise and fall.
Follower motion with modified uniform velocity:
It is observed in the displacement diagrams of the follower with uniform velocity that the acceleration of the follower becomes infinite at the beginning and ending of rise and return strokes. In order to prevent this, the displacement diagrams are slightly modified. In the modified form, the velocity of the follower changes uniformly during the beginning and end of each stroke. Accordingly, the displacement of the follower varies parabolically during these periods. With this modification, the acceleration becomes constant during these periods, instead of being infinite as in the uniform velocity type of motion. The displacement, velocity and acceleration patterns shown in fig 3.9

(a) Displacement diagram
(b) Velocity diagram
(c) Follower motion with uniform
acceleration and retardation (UARM):

Here, the displacement of the follower varies parabolically with respect to angular displacement of cam. Accordingly, the velocity of the follower varies uniformly with respect to angular displacement of cam. The acceleration/retardation of the follower becomes constant accordingly. The displacement, velocity and acceleration patterns are shown fig 3.10
(d) Simple Harmonic Motion: In fig3.11, the motion executed by point Pl, which is the projection of point P on the vertical diameter is called simple harmonic motion. Here, P moves with uniform angular velocity $\omega_p$, along a circle of radius $r\ (r = s/2)$. 

![Diagram of Simple Harmonic Motion](image-url)
Fig 3.11

Cycloidal motion:

Cycloid is the path generated by a point on the circumference of a circle, as the circle rolls without slipping, on a straight/flat surface. The motion executed by the follower here, is similar to that of the projection of a point moving along a cycloidal curve on a vertical line as shown in figure 6.12.
(a) Displacement diagram

Angle of outstroke ($\theta_o$)

(b) Velocity diagram

$c_o$ (velocity max)

(c) Acceleration diagram

$\theta_o/4$
(1) **Draw the cam profile for following conditions:**

Follower type = Knife edged, in-line; lift = 50mm; base circle radius = 50mm; out stroke with SHM, for $60^\circ$ cam rotation; dwell for $45^\circ$ cam rotation; return stroke with SHM, for $90^\circ$ cam rotation; dwell for the remaining period.

(2) Draw the cam profile for the same operating conditions of with the follower off set by 10 mm to the left of cam center.

**Displacement diagram:**

Cam profile:
Cam profile with 10 mm offset:

(2) Draw the cam profile for following conditions:

Follower type = roller follower, in-line; lift = 25mm; base circle radius = 20mm; roller radius = 5mm; out stroke with Uniform acceleration and retardation, for 120° cam rotation; dwell for 60° cam rotation; return stroke with Uniform acceleration and retardation, for 90° cam rotation; dwell for the remaining period.

(4) Draw the cam profile for conditions same with follower off set to right of cam center by 5mm and cam rotating counter clockwise.
Displacement Diagram:

Cam profile:
(3) Draw the cam profile for following conditions:

Follower type = knife edged follower, in line; lift = 30mm; base circle radius = 20mm; out stroke with uniform velocity in $120^0$ of cam rotation; dwell for $60^0$; return stroke with uniform velocity, during $90^0$ of cam rotation; dwell for the remaining period.
Displacement Diagram

Cam profile
(4) Draw the cam profile for following conditions:

Follower type = flat faced follower, in line; follower rises by 20mm with SHM in $120^\circ$ of cam rotation, dwells for $30^\circ$ of cam rotation; returns with SHM in $120^\circ$ of cam rotation and dwells during the remaining period. Base circle radius = 25mm.

Displacement Diagram:

Cam profile
4. Gears

Introduction: The slip and creep in the belt or rope drives is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slip is to reduce the velocity ratio of the drive. In precision machines, in which a definite velocity ratio is important (as in watch mechanism, special purpose machines..etc), the only positive drive is by means of gears or toothed wheels.

Gears are machine elements that transmit motion by means of successively engaging teeth. The gear teeth act like small levers. Gears are highly efficient (nearly 95%) due to primarily rolling contact between the teeth, thus the motion transmitted is considered as positive.

Gears essentially allow positive engagement between teeth so high forces can be transmitted while still undergoing essentially rolling contact. Gears do not depend on friction and do best when friction is minimized.

4.1 Gear Classification: Gears may be classified according to the relative position of the axes of revolution. The axes may be

1. Gears for connecting parallel shafts,
2. Gears for connecting intersecting shafts,
3. Gears for neither parallel nor intersecting shafts.

Gears for connecting parallel shafts

1. Spur gears: Spur gears are the most common type of gears. They have straight teeth, and are mounted on parallel shafts. Sometimes, many spur gears are used at once to create very large gear reductions. Each time a gear tooth engages a tooth on the other gear, the teeth collide, and this impact makes a noise. It also increases the stress on the gear teeth. To reduce the noise and stress in the gears, most of the gears in your car are helical.

Spur gears are the most commonly used gear type. They are characterized by teeth, which are perpendicular to the face of the gear. Spur gears are most commonly available, and are generally the least expensive.
Limitations: Spur gears generally cannot be used when a direction change between the two shafts is required.

Advantages: Spur gears are easy to find, inexpensive, and efficient.

2. Parallel helical gears: The teeth on helical gears are cut at an angle to the face of the gear. When two teeth on a helical gear system engage, the contact starts at one end of the tooth and gradually spreads as the gears rotate, until the two teeth are in full engagement.

This gradual engagement makes helical gears operate much more smoothly and quietly than spur gears. For this reason, helical gears are used in almost all car transmission. Because of the angle of the teeth on helical gears, they create a thrust load on the gear when they mesh. Devices that use helical gears have bearings that can support this thrust load.

One interesting thing about helical gears is that if the angles of the gear teeth are correct, they can be mounted on perpendicular shafts, adjusting the rotation angle by 90 degrees. Helical gears to have the following differences from spur gears of the same size:

Tooth strength is greater because the teeth are longer. Greater surface contact on the teeth allows a helical gear to carry more load than a spur gear. The longer surface of contact reduces the efficiency of a helical gear relative to a spur gear.

Rack and pinion (The rack is like a gear whose axis is at infinity.): **Racks** are straight gears that are used to convert rotational motion to translational motion by means of a gear mesh. (They are in theory a gear with an infinite pitch Diameter). In theory, the torque and angular velocity of the pinion gear are related to the Force and the velocity of the rack by the radius of the pinion gear, as is shown. Perhaps the most well-known application of a rack is the rack and pinion steering system used on many cars in the past.
Gears for connecting intersecting shafts: Bevel gears are useful when the direction of a shaft's rotation needs to be changed. They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well.

The teeth on bevel gears can be straight, spiral or hypoid. Straight bevel gear teeth actually have the same problem as straight spur gear teeth, as each tooth engages; it impacts the corresponding tooth all at once. Just like with spur gears, the solution to this problem is to curve the gear teeth. These spiral teeth engage just like helical teeth: the contact starts at one end of the gear and progressively spreads across the whole tooth.

On straight and spiral bevel gears, the shafts must be perpendicular to each other, but they must also be in the same plane. The hypoid gear, can engage with the axes in different planes.

Neither parallel nor intersecting shafts: Helical gears may be used to mesh two shafts that are not parallel, although they are still primarily use in parallel shaft applications. A special application in which helical gears are used is a crossed gear mesh, in which the two shafts are perpendicular to each other.

worm gear: Worm gears are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater.
**TERMINOLOGY:**

**Addendum:** The radial distance between the Pitch Circle and the top of the teeth.

**Arc of Action:** Is the arc of the Pitch Circle between the beginning and the end of the engagement of a given pair of teeth.

**Arc of Approach:** Is the arc of the Pitch Circle between the first point of contact of the gear teeth and the Pitch Point.

**Arc of Recession:** That arc of the Pitch Circle between the Pitch Point and the last point of contact of the gear teeth.

**Backlash:** Play between mating teeth.

**Base Circle:** The circle from which is generated the involute curve upon which the tooth profile is based.

**Center Distance:** The distance between centers of two gears.

**Chordal Addendum:** The distance between a chord, passing through the points where the Pitch Circle crosses the tooth profile, and the tooth top.

**Chordal Thickness:** The thickness of the tooth measured along a chord passing through the points where the Pitch Circle crosses the tooth profile.

**Circular Pitch:** Millimeter of Pitch Circle circumference per tooth.
**Circular Thickness**: The thickness of the tooth measured along an arc following the Pitch Circle.

**Clearance**: The distance between the top of a tooth and the bottom of the space into which it fits on the meshing gear.

**Contact Ratio**: The ratio of the length of the Arc of Action to the Circular Pitch.

**Dedendum**: The radial distance between the bottom of the tooth to pitch circle.

**Diametral Pitch**: Teeth per mm of diameter.

**Face**: The working surface of a gear tooth, located between the pitch diameter and the top of the tooth.

**Face Width**: The width of the tooth measured parallel to the gear axis.

**Flank**: The working surface of a gear tooth, located between the pitch diameter and the bottom of the teeth.

**Gear**: The larger of two meshed gears. If both gears are the same size, they are both called "gears".

**Land**: The top surface of the tooth.

**Line of Action**: That line along which the point of contact between gear teeth travels, between the first point of contact and the last.

**Module**: Millimeter of Pitch Diameter to Teeth.

**Pinion**: The smaller of two meshed gears.

**Pitch Circle**: The circle, the radius of which is equal to the distance from the center of the gear to the pitch point.

**Diametral pitch**: Teeth per millimeter of pitch diameter.

**Pitch Point**: The point of tangency of the pitch circles of two meshing gears, where the Line of Centers crosses the pitch circles.

**Pressure Angle**: Angle between the Line of Action and a line perpendicular to the Line of Centers.

**Profile Shift**: An increase in the Outer Diameter and Root Diameter of a gear, introduced to lower the practical tooth number or achieve a non-standard Center Distance.

**Ratio**: Ratio of the numbers of teeth on mating gears.
**Root Circle:** The circle that passes through the bottom of the tooth spaces.

**Root Diameter:** The diameter of the Root Circle.

**Working Depth:** The depth to which a tooth extends into the space between teeth on the mating gear.

**Fundamental Law of Gear-Tooth**

Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The above expression is the fundamental law of gear-tooth action.

**Formation of teeth:**

Involute teeth

Cycloidal teeth

**Involute curve:**

The curve most commonly used for gear-tooth profiles is the involute of a circle. This **involute curve** is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string, which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the **base circle**.
Cycloidal Curve

Consider a pinion driving wheel as shown in figure. When the pinion rotates in clockwise, the contact between a pair of involute teeth begins at $K$ (on the near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at $L$ (outer end of the tooth face on the pinion or on the flank near the base circle of wheel).

$MN$ is the common normal at the point of contacts and the common tangent to the base circles. The point $K$ is the intersection of the addendum circle of wheel and the common tangent. The point $L$ is the intersection of the addendum circle of pinion and common tangent.
The length of path of contact is the length of common normal cut-off by the addendum circles of the wheel and the pinion. Thus the length of part of contact is $KL$ which is the sum of the parts of path of contacts $KP$ and $PL$. Contact length $KP$ is called as path of approach and contact length $PL$ is called as path of recess.

Path of approach: $KP$

\[
KP = KN − PN = \sqrt{(R_4)^2 - R^2 \cos^2 \phi - R \sin \phi}
\]

Path of recess: $PL$

\[
PL = ML − MP = \sqrt{(r_a)^2 - r^2 \cos^2 \phi - r \sin \phi}
\]

Length of path of contact:

\[
KL = KP + PL = \sqrt{(R_4)^2 - R^2 \cos^2 \phi + \sqrt{(r_a)^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi}
\]

**Arc of contact:** Arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Figure, the arc of contact is $EPF$ or $GPH$.

The arc $GP$ is known as arc of approach and the arc $PH$ is called arc of recess. The angles subtended by these arcs at O1 are called angle of approach and angle of recess respectively.

Length of arc of approach = arc $GP$

\[
\text{Length of arc of approach} = \frac{\text{Length of path of approach}}{\cos \phi} = \frac{KP}{\cos \phi}
\]

Length of arc of recess = arc $PH$

\[
\text{Length of arc of recess} = \frac{\text{Length of path of recess}}{\cos \phi} = \frac{PL}{\cos \phi}
\]

Length of arc contact = arc $GPH$

\[
\text{Length of arc contact} = \text{arc } GP + \text{arc } PH = \frac{KP}{\cos \phi} + \frac{PL}{\cos \phi} = \frac{KL}{\cos \phi} = \frac{\text{Length of path of contact}}{\cos \phi}
\]
Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch.

\[
\text{Contact ratio} = \frac{\text{Length of the arc of contact}}{P_c}
\]

\[P_c = \text{Circular pitch} = \pi \times m\quad \text{and} \quad m = \text{Module.}\]

Continuous motion transfer requires two pairs of teeth in contact at the ends of the path of contact, though there is only one pair in contact in the middle of the path, as in Figure. The average number of teeth in contact is an important parameter - if it is too low due to the use of inappropriate profile shifts or to an excessive centre distance. The manufacturing inaccuracies may lead to loss of kinematic continuity - that is to impact, vibration and noise The average number of teeth in contact is also a guide to load sharing between teeth; it is termed the contact ratio.
The tooth tip of the pinion will then undercut the tooth on the wheel at the root and damages part of the involute profile. This effect is known as *interference*, and occurs when the teeth are being cut and weakens the tooth at its root.

In general, the phenomenon, when the tip of tooth undercuts the root on its mating gear is known as interference. Similarly, if the radius of the addendum circles of the wheel increases beyond O2M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called interference points.

Interference may be avoided if the path of the contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O1N and of the wheel is O2M.

The interference may only be prevented, if the point of contact between the two teeth is always on the involute profiles and if the addendum circles of the two mating gears cut the common tangent to the base circles at the points of tangency.

**Methods to avoid Interference**

1. Height of the teeth may be reduced.

2. Under cut of the radial flank of the pinion.

3. Centre distance may be increased. It leads to increase in pressure angle.

4. By tooth correction, the pressure angle, centre distance and base circles remain unchanged, but tooth thickness of gear will be greater than the pinion tooth thickness.

**Minimum numbers of teeth on the pinion avoid Interference**

The pinion turns clockwise and drives the gear as shown in Figure.

Points M and N are called interference points. i.e., if the contact takes place beyond M and N, interference will occur.

The limiting value of addendum circle radius of pinion is O1N and the limiting value of addendum circle radius of gear is O2M. Considering the critical addendum circle radius of gear, the limiting number of teeth on gear can be calculated.

The equation gives minimum number of teeth required on the pinion to avoid interference:

\[
t = \frac{2a_p}{\left(1 + G(G + 2)\sin^2 \phi\right)^{\frac{1}{2}} - 1}
\]
If the number of teeth on pinion and gear is same: \( G = 1 \)

\[
t = \frac{2a_p}{\left( \frac{1}{2} \left( 1 + 3 \sin^2 \phi \right) \right)^{\frac{1}{2}} - 1}
\]

The equation gives minimum number of teeth required on the wheel to avoid interference.

\[
a_w, m = \frac{mT}{2} \left[ \left( 1 + \frac{t}{T} \left( \frac{t}{T} + 2 \right) \sin^2 \phi \right)^{\frac{1}{2}} - 1 \right]
\]

\[
a_w = \frac{T}{2} \left[ \left( 1 + \frac{t}{T} \left( \frac{t}{T} + 2 \right) \sin^2 \phi \right)^{\frac{1}{2}} - 1 \right]
\]

\[
T = \frac{2a_w}{\left( 1 + \frac{1}{G} \left( \frac{1}{G} + 2 \right) \sin^2 \phi \right)^{\frac{1}{2}} - 1}
\]

Gears Trains
A gear train is two or more gear working together by meshing their teeth and turning each other in a system to generate power and speed. It reduces speed and increases torque. To create large gear ratio, gears are connected together to form gear trains. They often consist of multiple gears in the train. The most common of the gear train is the gear pair connecting parallel shafts. The teeth of this type can be spur, helical or herringbone.

The angular velocity is simply the reverse of the tooth ratio. Any combination of gear wheels employed to transmit motion from one shaft to the other is called a gear train. The meshing of two gears may be idealized as two smooth discs with their edges touching and no slip between them. This ideal diameter is called the Pitch Circle Diameter (PCD) of the gear.

**Simple Gear Trains**

The typical spur gears as shown in diagram. The direction of rotation is reversed from one gear to another. It has no affect on the gear ratio. The teeth on the gears must all be the same size so if gear A advances one tooth, so does B and C.

\[
\begin{align*}
t &= \text{number of teeth on the gear}, \\
D &= \text{Pitch circle diameter}, \quad N = \text{speed in rpm} \\
m &= \text{module} = \frac{D}{t} \\
\text{and} \\
\text{module must be the same for all gears otherwise they would not mesh.}
\end{align*}
\]

\[
\begin{align*}
m &= \frac{D_A}{t_A} = \frac{D_B}{t_B} = \frac{D_C}{t_C} \\
D_A &= m t_A; \quad D_B = m t_B \quad \text{and} \quad D_C = m t_C \\
\omega &= \text{angular velocity}. \\
v &= \text{linear velocity on the circle.} \quad v = \omega \frac{D}{2} = \omega r
\end{align*}
\]

\[
\begin{align*}
v &= \omega_A \frac{D_A}{2} = \omega_B \frac{D_B}{2} = \omega_C \frac{D_C}{2} \\
\omega_A D_A &= \omega_B D_B = \omega_C D_C \\
\omega_A m t_A &= \omega_B m t_B = \omega_C m t_C \\
\omega_A t_A &= \omega_B t_B = \omega_C t_C
\end{align*}
\]

or in terms of \( \text{rev/min} \)

\[
N_A t_A = N_B t_B = N_C t_C
\]
**Compound Gear train**

Compound gears are simply a chain of simple gear trains with the input of the second being the output of the first. A chain of two pairs is shown below. Gear B is the output of the first pair and gear C is the input of the second pair. Gears B and C are locked to the same shaft and revolve at the same speed. For large velocities ratios, compound gear train arrangement is preferred.
Reverted gear train:

The driver and driven axes lies on the same line. These are used in speed reducers, clocks and machine tools. If \( R \) and \( T \)=Pitch circle radius & number of teeth of the gear.

\[
\frac{\omega_A}{t_B} = \frac{\omega_B}{t_A} \quad \text{and} \quad \frac{\omega_C}{t_D} = \frac{\omega_D}{t_C}
\]

\[
\omega_A = \frac{t_B \times \omega_B}{t_A} \quad \text{and} \quad \omega_C = \frac{t_D \times \omega_D}{T_C}
\]

\[
\omega_A \times \omega_C = \frac{t_B \times \omega_B}{t_A} \times \frac{t_D \times \omega_D}{t_C}
\]

\[
\omega_B \times \omega_D = \frac{t_B}{t_A} \times \frac{t_D}{t_C}
\]

\[
GR = \frac{N_A}{N_D} = \frac{t_B \times t_D}{t_A \times t_C}
\]

\[
R_A + R_B = R_C + R_D \quad \text{and} \quad t_A + t_B = t_C + t_D
\]
UNIT 5 FRICTION

FRICTION

Dry friction – Friction in screw jack – Pivot and collar friction - Plate clutches - Belt and rope drives - Block brakes, band brakes.

Friction:

The opposing force which acts in the opposite direction of the movement of the upper block is called the force of friction or friction.

Types of friction:

1. Static friction: It is experienced by a body, when at rest.
2. Dynamic friction: It is friction experienced by a body when in motion.
   a. Sliding friction: It is friction experienced by a body when it slides over another body.
   b. Rolling friction: It is friction experienced between the surfaces which the balls or rollers interposed between them.
   c. Pivot friction: It is the friction experienced by a body due to motion of rotation.

Further classified

1. Friction between unlubricated surfaces
2. Friction between lubricated surfaces.

Laws of dry or solid friction:

- The force of friction directly proportional to the normal load between the surfaces.
- The force of friction is independent of the area of the contact surface for a given normal load.
- The force of friction depends upon material which the contact surfaces or made.
- The force of friction is independent of the velocity of sliding of one body relative to other body.
Coefficient of friction ($\mu$):

It is as the ratio the limiting friction ($F$) to the normal reaction ($R_N$) between the two bodies.

$$\mu = \frac{F}{R_N}$$

Angle of friction:

It may be defined as the angle which the resultant reaction $R$ makes with normal reactions

$$\tan \phi = \frac{F}{R_N}$$

Screw jack:

The screw jack is a device lifting loads. For lifting heavy loads by applying a comparatively smaller effort at its handle. The principle on which a screw jack works is smaller to that of an inclined plane.

Torque required lifting the load by a screw jack

Let $p =$ pitch of the screw

$d =$ mean diameter of the screw

$\alpha =$ helix angle
P = effort applied at the circumference of the screw to lift the load

W = load to be lifted

μ = coefficient of the friction

\[
\tan \alpha = \frac{p}{\pi d}
\]

Torque required overcoming friction between the screw and nut

\[
T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}
\]

The torque required to overcome friction at the collar

\[
T_2 = \mu_1 W \left( \frac{R_1 + R_2}{2} \right) = \mu_1 W.R
\]

Total Torque required overcoming friction

\[
T = T_1 + T_2 = P \times \frac{d}{2} + \mu_1 W.R
\]

If an \( P_1 \) is applied at the end of a lever of arm \( l \), then the total torque required to overcome friction must be equal to the torque applied at the end of the lever

\[
T = P \times \frac{d}{2} = P_1 l
\]
2. Torque required lower the load by a screw jack

Let  
\[ p = \text{pitch of the screw} \]
\[ d = \text{mean diameter of the screw} \]
\[ \alpha = \text{helix angle} \]
\[ P = \text{effort applied at the circumference of the screw to lift the load} \]
\[ W = \text{load to be lifted} \]
\[ \mu = \text{coefficient of the friction} \]

\[ \tan \alpha = \frac{p}{\pi d} \]

Torque required overcoming friction between the screw and nut

\[ T = P \times \frac{d}{2} = W \tan (\phi - \alpha) \frac{d}{2} \]

Efficiency of the screw jack:

The efficiency of the screw jack may be defined as the ratio between the ideal efforts to actual effort.

\[ \therefore \text{Efficiency, } \eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W \tan \alpha}{W \tan (\alpha + \phi)} = \frac{\tan \alpha}{\tan (\alpha + \phi)} \]
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The efficiency of the screw jack may also be defined as the ratio of mechanical advantage to the velocity ratio.

We know that mechanical advantage,
\[
M.A = \frac{W}{P} = \frac{W \times 2l}{P \times d} = \frac{W \times 2l}{W \tan (\alpha + \phi) d} = \frac{2l}{\tan (\alpha + \phi) d}
\]

and velocity ratio,
\[
V.R = \frac{\text{Distance moved by the effort (} P \text{), in one revolution}}{\text{Distance moved by the load (} W \text{), in one revolution}} = \frac{2 \pi l}{\tan \alpha \times \pi d} = \frac{2 l}{\tan \alpha \times d}
\]

\[
\therefore \text{ Efficiency, } \eta = \frac{M.A}{V.R} = \frac{2l}{\frac{\tan (\alpha + \phi) d}{\tan \alpha d}} = \frac{2l}{\tan (\alpha + \phi)}
\]

Self locking and overhauling of screws

Torque required to lower the load

\[
T = P \times \frac{d}{2} = W \tan (\phi - \alpha) \frac{d}{2}
\]

In the above expressions, if \( \phi < \alpha \), then the torque required to lower the load will be negative, the load will start moving downward without application of force, such a condition is known as overhauling of screw.

If \( \phi > \alpha \), then the torque required to lower the load will be positive, indicating that an effort applied lower the load, such a condition is known as self locking of screw.
BELT

The belt or ropes are used to transmit power from one shaft to another shaft by means of pulleys which rotate at the same speed or at different speed.

Types of belt drives.

1. Light drives: belt speed up to 10m/s
2. Medium drives: speed 10m/s to 22 m/s
3. Heavy drives: Speed over 22m/s

Types of belt

1. Flat belt
2. V- belt
3. Circular belt or rope

Type of flat belt drives

Open belt drive
Cross belt drive
Quarter turn belt drive
Belt drive with idler pulley

Velocity of belt drive

It is the ratio between the velocities of the driver and follower or driven.

\[ \frac{d_1}{d_2} = \text{diameter of the driver} \]
\[ \frac{N_1}{N_2} = \text{diameter of the follower} \]
\[ \frac{N_1}{N_2} = \text{Speed of the driver r.p.m} \]
\[ \frac{N_1}{N_2} = \text{Speed of the driven r.p.m} \]
Let $r_1$ and $r_1 = \text{Radii of the larger or smaller pulleys}$

$x = \text{Distance between the centres of the two pulleys}$

$L = \text{Total length of the belt.}$

Length of an open belt drive

Length of the cross belt drive.

Power transmitted by a belt

$T_1$ and $T_2 = \text{Tensions in the tight and slack side of the belt respectively N}$

$r_1$ and $r_1 = \text{Radii of the larger or smaller pulleys}$

$v = \text{Velocity of the belt m/s}$

Power $= (T_1-T_2)v \text{ W}$

Ratio of driving tensions for flat belt drive

$\mu$

$\theta = \text{angle of contact}$

determination of angle of contact